Grade 6/7

Number Sense

Assessment Package

SNAP Assessment

First Steps in Math

Diagnostic Assessments & Learning Activities

SD71-Comox Valley Schools, 2004

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SD71 4-8 Numeracy Assessment

Intermediate/Middle Years Foundations

Intermediate and Middle Years teachers identify and monitor their learners' strengths and needs to ensure continuous growth in numeracy. This assessment tool is designed for classroom teachers and focuses on critical skills that foster numeracy development. The tool can be used to inform instructional decisions as part of the ongoing instructional cycles throughout the Intermediate and Middle Years and is useful for school data collection to determine school goals and to pass on to future teachers in support of transitions.

Wiliam and Leahy (2015, p.9) state that many in education talk of 'data driven instruction' resulting in large scale assessments that provide information on our learners after the fact – too late to do anything about it! We should instead be focusing on 'decision-driven data collection' answering the questions:

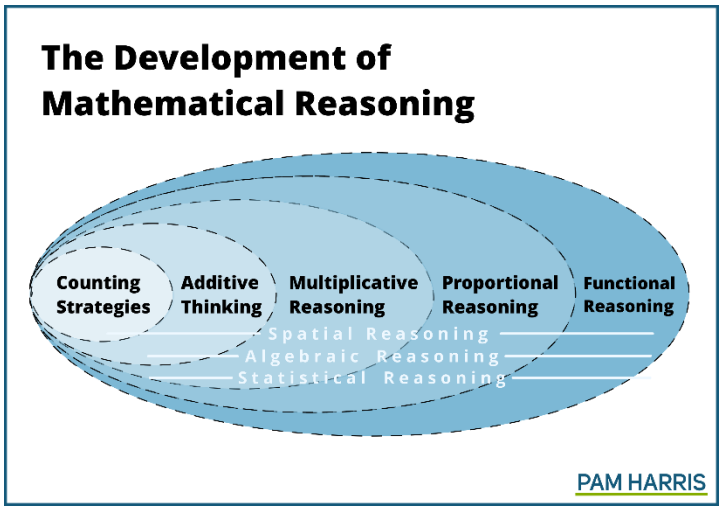
- "What do you want to know about your learners?"
- "When do you want to know it?"

We need 'laser-beam focused' assessments to help us reach all our learners that are quick to implement, interpret and act upon.

NOTE: Please note that these assessment tools focus on some of the key numeracy areas and do not represent a comprehensive numeracy learning program. For more information about our district's holistic approach to numeracy learning, please see the SD71 Numeracy Framework.

Grades 4-7 Numeracy Developmental Stages:

- ✓ Grade 4 students begin to move from the world of additive thinking into multiplicative thinking.
- ✓ Students need to recognize multiplication in repeating equal quantities, rates, ratios, arrays, and as a product of measures.
- ✓ They see division as the inverse of multiplication and as both partitive and quotative.



- ✓ They are continuing to develop their basic facts as they find and use strategies for multiplication and division to 100.
- ✓ The world of fractions looms large on the landscape of learning and students need to think flexibly about fractions of area, measurement, and sets. They will name, order, and compare fractions.
- ✓ *Place Value* expands infinitely now in both directions, and there are special fractions, called 'decimal fractions' that are also part of the place value system.
- ✓ They also learn to operate with fractions (add, subtract, multiply and divide).
- ✓ They will take all these concepts into solving numeracy tasks based on real-world experiences.

What's on the Horizon?

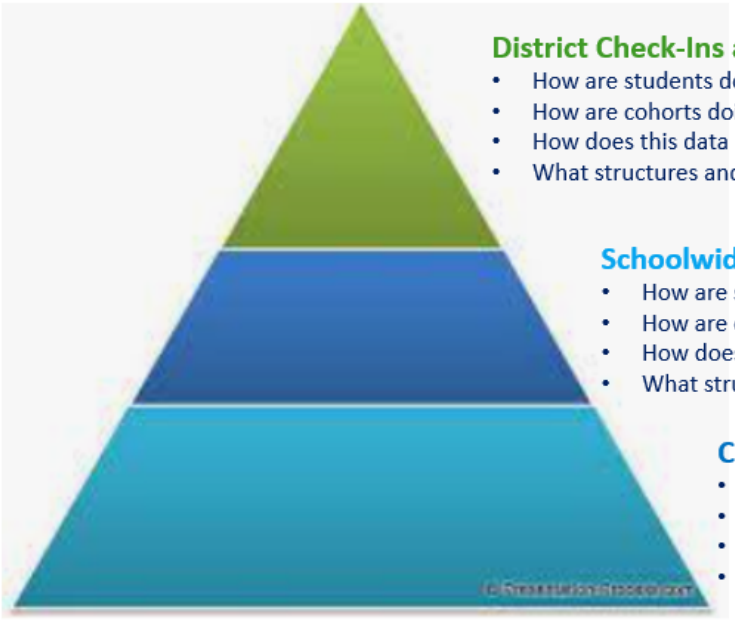
- ✓ In **Grades 8 and 9** students build on their understanding of fractions and decimals, moving flexibly between fractions, decimals, ratios and percents, and operating with them.
- ✓ They will use their developed multiplicative understanding to find perfect squares, cubes, square roots, and cube roots so having *automaticity* with basic facts is crucial.
- ✓ Students also operate with integers and polynomials and begin to work with exponents.

Mathematics is not about numbers, equations, computations, or algorithms: it is about understanding.

William Paul Thurston

Assessment Design for the Frameworks

Grounded in Classroom Assessment System



District Check-Ins and Provincial Assessments

- How are students doing across our system?
- How are cohorts doing across time?
- How does this data inform our district's long term goals and plans?
- What structures and strategies are needed for continuous improvement?

Schoolwide Assessment

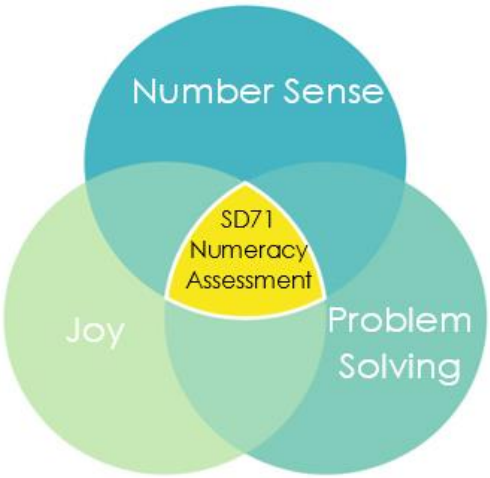
- How are students doing across the school?
- How are cohorts doing across the grades?
- How does this data inform our school goals and plans?
- What structures and strategies are needed to address needs?

Classroom Assessment

- How is each student doing?
- How is the class doing?
- Who needs supplemental learning?
- Did supplemental learning make a difference?

District Numeracy Check-In Points

With in the SD71 Numeracy Framework, there are district check-in points to monitor how learners in our system are doing across the years and to inform decision making regarding numeracy initiatives, professional learning opportunities, and resources. The district will be extracting the data from both the SNAP and Problem-Solving Assessments that classroom teachers have entered in Grades 3, 5, and 8.



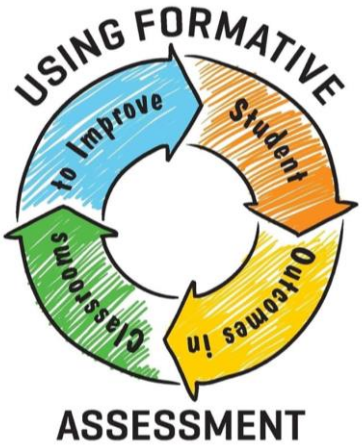
District Check-In Points and Provincial Assessments Timeline:

Number Sense Numeracy Assessments:

1. **Whole class**: SNAP (Student Numeracy Assessment and Practice), every 6-8 weeks. (Fall/Winter/Spring)

2. **Targeted Diagnostic Assessment**: First Steps in Math - Targets misconceptions and gaps, accompanied with learning activities to build key math understandings for the whole class, small groups, and/or one-to-one instruction.

3. **Daily Observations**: Can be recorded in checklists, comments, and reflections.

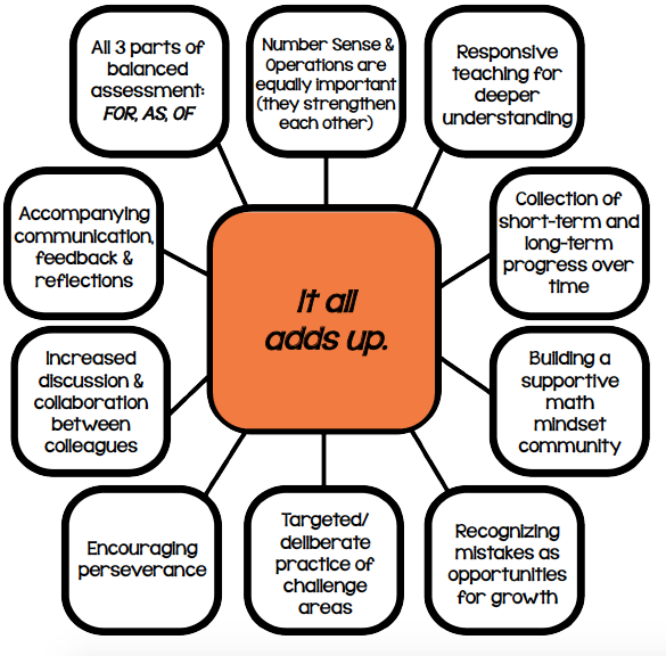


We value what we measure.

Do we measure what we value?

Whole Class Assessment SNAP Grades 4-7:

The Student Numeracy Assessment and Practice (SNAP) is the Chilliwack School District's numeracy assessment for all students in grades K – 7. It was created by a group of Chilliwack educators and has been used in all grades K – 7 classes since September 2016. The SNAP is a unique assessment; not only is it a measurement of achievement, but it is intended to be used as a practice tool throughout the entire year. The data it provides should be used to inform and guide instructional planning.



The SNAP is a two-page assessment that focuses on the foundational skills of mathematics: Number Sense and Operations. It **compliments any balanced math program** and quickly provides teachers the information they need for responsive planning and instruction.

SNAP is fully aligned with the BC Curricular Competencies in math. Each area of the assessment is connected to a particular competency, and the competencies are built right into the rubric.

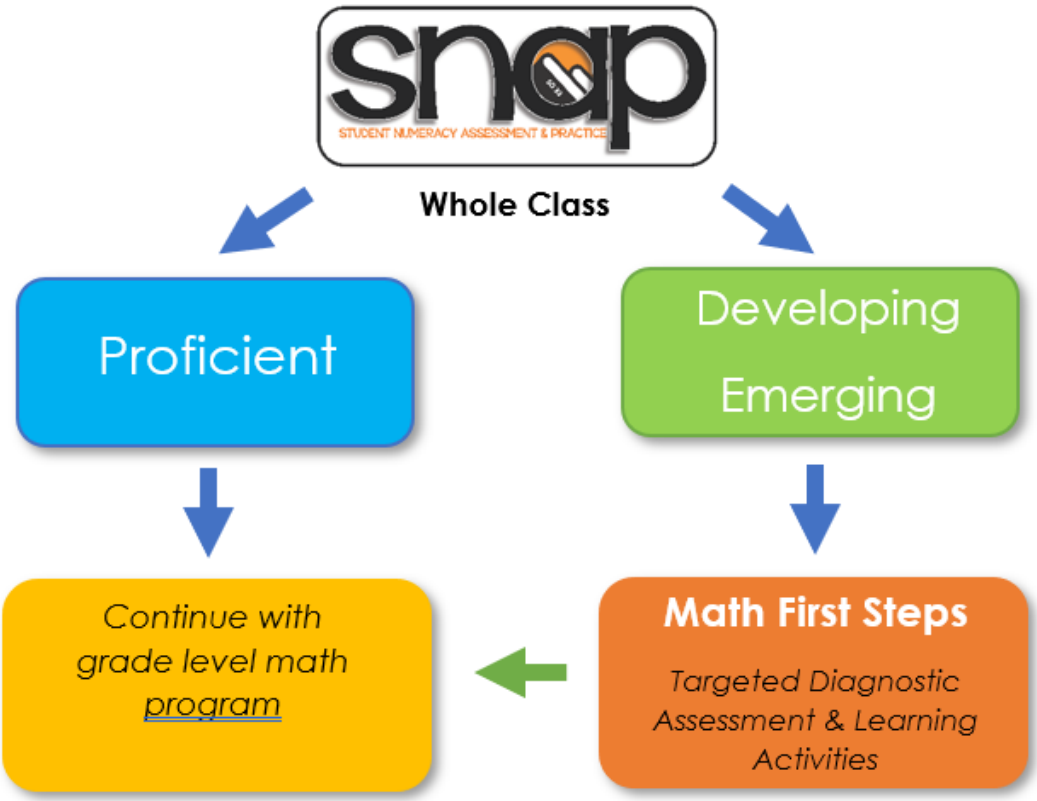
(SNAP Teacher Guide – updated, Chilliwack School District)

NOTE: Please reference the SNAP website for teacher guides, templates, rubrics, and exemplars for your grade.

Identifying who may require targeted instruction

Use the SNAP with the **whole class** and sort completed assessments into 2-3 groups – Students who have demonstrated ***mastery/proficiency*** and those who have not. You may want to break the second group down into 'developing' and 'emerging' groupings if appropriate.

SNAP Grade 4: • Number Sense 0 – 10 000 • Operations: Multiplication
SNAP Grade 5: • Number Sense 0 – 1 000 000 • Operations: Division
SNAP Grade 6: • Thousandths to Billions • Decimal division
SNAP Grade 7: • Integers • Fractions/Decimals/Percent



Targeted Diagnostic Assessment- First Steps in Math

It is this second group (emerging/developing) with whom we use the set of rich ***First Steps in Math - Number Sense*** diagnostics to identify learners' misconceptions and gaps and to determine which learning activities can address these and build their understanding. The *First Steps in Math* resource is designed to pinpoint and target student misconceptions and can be used to inform next steps in learning. This package includes the curated diagnostic tasks from the resource.

It is recommended that the teacher focus on **one** diagnostic assessment at a time, followed by an analysis to determine the next step for supplemental learning. "Next step" learning activities that correspond with each diagnostic are also provided to target the misconception or key understanding. All activities on the *Learning Activities* sheet will support the same misconception starting with less complex activities moving toward more complex activities. If the suggested learning activity does not seem to work for learners, try another corresponding activity. Professional judgement and relationship with your students will guide you in determining what works better for them.

Note that both the diagnostic assessments and learning activities can benefit the whole class or can be done during small group and/or one-to-one instruction.

Frequently Asked Questions:

1. ***Can I have my LST or CST assess my students for me?***

The SNAP and First Steps in Math numeracy assessments are designed for classroom teachers to use as part of their teaching, learning and assessment cycle. It is important for teachers to know where their students' strengths and stretches are in key numeracy skills to inform planning and to scaffold learning to meet the needs of their learners.

2. ***Do I have to assess students all at once?***

Teachers assess at a variety of times in a variety of ways, depending on assessment purposes and what works for their students. As described herein, the SNAP can be performed as a whole class, while First Steps in Math resources can help to identify specific misconceptions and gaps as well as provide suggestions for targeted instruction to address them.

3. ***Do I have to use this assessment if I have my own numeracy assessment tools?***

Teacher colleagues in our district have reviewed and discussed many possible strategies and assessments for number sense learning and have selected SNAP and First Steps in Math based on research and what works with students. It is recommended that intermediate and middle years teachers become familiar with the common language and understandings of these district wide assessment tools, as they will be used throughout K-9, and support student numeracy learning across the school years.

4. ***What do I do if my students are not demonstrating proficiency on a skill?***

Teaching, Learning and Assessment Cycle:

i. *Whole class teaching, and repeated experience practicing the skill*
ii. *Assess*
iii. *Targeted classroom instruction according to identified student needs.*
iv. *Assess*
v. *Targeted classroom instruction and/or consult with school-based team about other interventions that may be necessary.*
vi. *Assess*

Frequently Asked Questions:

6. ***How will the information collected from this assessment be used?***

Classroom Teachers *use this information to inform instruction and monitor progress of their students' numeracy skills. Additionally, this information will be helpful to pass on to the next year's teacher.*

The School *can use the data for school growth plans, allocation of resources and to inform school-based team discussions and strategies.*

The District *will collect the SNAP- Number Sense and Operations information in Grades 3, 5, and 8. This data will be used to inform decisions regarding resource allocation and professional learning opportunities, as well as to monitor the numeracy development of student cohorts over time.*

Bibliography

First Steps in Mathematics. (2007-13). Pearson Canada.

SD38 - Chilliwack School District (retrieved 2024). *SNAP: Student Numeracy Assessment and Practice*.

Wiliam, D. and Leahy, S. (2015). *Embedding formative assessment: Practical techniques for K-12 classrooms* Learning Sciences International.

Grade 6-7
SNAP Assessment

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TEACHER GUIDE

What is the SNAP?

The Student Numeracy Assessment and Practice (SNAP) is the Chilliwack district numeracy assessment for all students in grades 2 – 7. It was created by a group of Chilliwack educators and has been used in all grades 2 – 7 classes since September 2016.

The SNAP is a unique assessment; not only is it a measurement of achievement, but it is **intended to be used as a practice tool throughout the entire year.** The data it provides should be used to **inform and guide instructional planning**. If only used as a summative assessment, the SNAP will not help in achieving one of our main goals, which is to improve students' proficiency in number sense and operations.

The SNAP is a two-page assessment that focuses on the foundational skills of mathematics: Number Sense and Operations. It compliments any balanced math program and quickly provides teachers the information they need for responsive planning and instruction. **Access the SNAP Number Sense and Operations templates under the SNAP Templates tab on the website.**

SNAP is fully aligned with the BC Curricular Competencies in math. Each area of the assessment is connected to a particular competency, and the competencies are built right into the grading rubric. **Access the grading rubrics under the SNAP Training tab on the website.** The rubrics are the same for all grades. It is a good idea to participate in collaborative marking with colleagues to help establish common expectations.

How to Effectively use the SNAP

SNAP practice does not always need to be on the SNAP templates; in fact, once areas of need are identified, most number sense and operations practice will happen through other strategies, such as daily high yield number sense routines (e.g. number talks, count around the circle) and whole or small-group instruction. **Find resources that support each of the four curricular competencies under the Resources tab on the website.** Explore the Recommended Links for sites that support the teaching and learning of number sense and operations.

Curricular Content and Competency Areas

While the SNAP templates and rubrics are the same for grades 2-7, the curricular content and competency goals (pulled directly from the BC Math Curriculum) change and follow a spiraled approach. The table below outlines the curricular areas that students will be assessed on at the end of May. The goal is that all students be proficient (3 on the rubric) in their grade-level standards by the end of the school year. The examples given in the Operations sections are examples of year-end appropriate operations. **There are no district-prescribed numbers or operations for the year-end assessment, but at the request of teachers, numbers and operations have been suggested below to provide guidance.**

Grade	Operations - Sample operations	Number Sense - Sample numbers	Number Sense – Skip counting sample numbers
2	Addition of two-digit numbers without regrouping 24+33 51+17	Number concepts to 100. Any two-digit number. 42 67	Count forward by: 2, 5, or 10 Count backward by: 2
3	Subtraction of three-digit numbers with regrouping 427-153 754-226	Number concepts to 1000. Any three-digit number. 327 568	Count forward by: 4 or 20 Count backward by: 3 or 5
4	Multiplication of a one-digit number by a three-digit number. 4x326 7x142	Number concepts to 10000. Any four-digit number. 5904 6138	Count forward by: 6s or 300s Count backward by: 4 or 25
5	Division of a three-digit number by a one-digit number with a remainder. 635÷3 291÷4	Number concepts to 1000000. Any six-digit number. 347075 762346	Count forward by: 7 or 250 Count backward by: 6 or 30
6	Division of four-digit decimal number to hundredths. Quotient should not exceed thousandths. 47.35÷5 71.76÷3	Number concepts thousandths to billions. Any decimal number to the thousandths. 45.892 534.21	Count forward by: .12 or 5000 Count backward by: .6 or 14
7	Percentage calculations. Find the percent of a number. Answer should be in the tenths or hundredths. 16% of 85 47% of 42	Integer concepts. Any negative two-digit whole number. -23 -75	Choose numbers that will make students count through 0. Count forward by: 4 or 12 Count backward by: 5 or 20

Remember that the SNAP templates are intended to be used throughout the year for any numbers or operations in your curriculum.

When introducing your students to the SNAP, take your time and explicitly teach and model each component of the assessment. Use content that the students should be confident with from previous years. You can chunk the assessment into smaller pieces. **The Zoom into SNAP templates under the Resources tab on the website chunk the assessment by competency**. You can complete SNAPs as a whole group guided activity and have students work with partners to help build confidence. Have students share their thinking; encourage them to use many different ways to demonstrate their thinking and solutions.

Remember that the SNAP templates are intended to be used throughout the year for any numbers or operations in your curriculum.

The SNAP templates

Access templates under SNAP Templates tab.

NUMBER SENSE:

See Grading Rubrics for specific criteria.

DRAW: The picture must show the value of the number. A written explanation or a legend should be included in the "write to describe your picture" box.

SKIP-COUNTING: Begin at the number and count forwards and backward by numbers chosen by the teacher. *Update – Spring 2024* Teachers have requested guidance on appropriate numbers to use in this section for the May assessment. We have provided sample numbers based on the curriculum at each grade in the table above.

EQUATIONS: Students who are demonstrating full proficiency will be using grade-appropriate operations in their equations. Teachers should be very specific about their expectations in this section to avoid students using equations like 4561+1=4562, for example (which is not a grade-appropriate operation in Gr. 4).

REAL-LIFE EXAMPLE: The examples must be realistic and specific. It is important that students demonstrate an understanding of value in their example. For instance, "Wayne Gretzky's number is 99" does not show an understanding of value; "we have 99 grade three students in our school" does. Literature and sharing out of real-life examples helps students to make connections to the numbers and add to their bank of knowledge. There is an excellent list of math picture books on the Coast Metro Elementary Math Project site.

NUMBER LINE: For grades 2-5, the endpoints to the number line are provided. For grades 6 & 7, the students choose their own endpoints according to the number chosen for the assessment. To demonstrate full proficiency, students will add at least three benchmarks to their number line to help situate the number. Clothesline Math is an excellent routine to help students to become more proficient with number lines.

REFLECTION: Reflections help increase the value of a learning experience. They allow students to link ideas and construct meaning from their experiences. Students should have opportunities to reflect on their learning at the end of every lesson. Explicit teaching about how to reflect effectively will improve the quality of student responses in this section; reflection sentence stems are available in the Connecting and Reflecting Resources page.

OPERATIONS

See Grading Rubrics for specific criteria.

ESTIMATE: Students will learn to value the skill of estimating through discussions about real-life situations where a person would typically estimate rather than calculate. In which situations would one prefer a high estimate? A low estimate? Explicit instruction on estimation strategies will allow students to select and use an appropriate strategy for the given operation.

DRAW: Students will visually represent the operation. The visual may or may not contain the solution to the operation. Consider the use of bar diagrams as an appropriate, proportional model for the operations. Simply replacing the numbers in the operation with a base ten representation does not demonstrate an understanding of the operation.

CALCULATE: Multiple grade-appropriate calculations demonstrate proficient achievement. Students are not required to use the standard algorithm for any operation. Using the reverse operation to "check" their work is also a recommended strategy. Refer to your grade-specific curriculum elaborations for suggested alternate computation strategies.

REAL-LIFE EXAMPLE OR WORD PROBLEM: Students will provide details on a real-life situation where the given operation would be used to find an amount. Look for evidence that communicates their understanding of the use of the operation. For example, if the operation was 316-141 a student could suggest, "there were 316 blueberries on the bush and I picked 141 of them." For the teacher to know if they understand what the difference between 316 and 141 represents in this situation, they should add, "How many blueberries were left on the bush?"

Grade 2 Math Story: Encourage students to draw pictures to "tell" their story if they do not have the written ability to write a short story. A quick follow up conversation will be required to know whether students are able to communicate their understanding.

REFLECTION: Reflections help increase the value of a learning experience. They allow students to link ideas and construct meaning from their experiences. Students should have opportunities to reflect on their learning at the end of every lesson. Explicit teaching about how to reflect effectively will improve the quality of student responses in this section; reflection sentence stems are available in the Connecting and Reflecting Resources page.

Data Entry

Chilliwack teachers will enter data by the end of November and by the end of May. November data entry is based on the previous year's outcomes, and is only to be completed by grades 3-7 teachers. For example, grade 4 teachers will assess their students at the beginning of the year based on the grade 3 target outcomes and using the grade 3 templates. All grades 2-7 teachers will enter data by the end of May based on the current year's outcomes.

Another unique feature of the SNAP is that students are scored by competency. You will not total or average their scores in the four competencies. Students have until the end of the school year to practice and become proficient at their grade-level learning standards, however if during your pre-assessments prior to May you have students fully proficient, you may enter their data and create learning extension opportunities for those students.

Exemplars

The exemplars on the website are intended to represent proficiency in all categories. We will be updating our exemplars on an ongoing basis. Please feel free to send in student samples that you believe clearly show student proficiency. Scan and send to joanne_britton@sd33.bc.ca.

Acknowledgements

We are grateful to the dedicated team of Chilliwack educators who crafted and piloted this assessment: *Christine Blessin, Jonathan Ferris, Kathy Isaac, Anna Lownie, Shannon McCann, Tammy McKinley, Kathleen Mitchell, Justin Moore, Kirk Savage, Paul Wojcik*

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SNAP

Number Sense (Thousandths to Billions)

Name: _____

Date: _____

Draw to represent the value of the number:

Write to describe your picture:

Write the number in expanded form:

Create 3 equations that equal the number:

Write a real life example that shows the value of the number:

Count **forwards** by _____ from the number.

Count **backwards** by _____ from the number.

Show where the number belongs on the number line.

Reflect:

Communicating & Representing: *Drawing, description, expanded form*	Understanding & Solving: *3 equations*	Connecting & Reflecting: *Real-life*	Reasoning & Analyzing: *Skip counting & number line*
1 2 3	1 2 3	1 2 3	1 2 3

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Operations Decimal Division SNAP

Name: _____

Date: _____

Operation: _____

Estimate – justify your thinking:

Represent - with a sketch or drawing:

Explain your sketch:

Calculate:

Write a Real Life Example or Word Problem: _____

_____

Reflect:

Communicating & Representing	Understanding & Solving	Connecting & Reflecting	Reasoning & Analyzing
1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4
Entire assessment	Draw and Calculate	Real-life & reflection	Estimate

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SNAP
Number Sense (Integers)

Name: _____

Date: _____

Draw to represent the value of the number:

Write to describe your picture:

Write the number in expanded form:

Create 3 equations that equal the number:

Write a real life example that shows the value of the number:

Count **forwards** by _____ from the number.

Count **backwards** by _____ from the number.

Show where the number belongs on the number line.

Reflect:

Communicating & Representing: *Drawing, description, expanded form*	Understanding & Solving: *3 equations*	Connecting & Reflecting: *Real-life*	Reasoning & Analyzing: *Skip counting & number line*
1 2 3	1 2 3	1 2 3	1 2 3

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Operations Fraction, Decimal, Percent SNAP

Name: _____

Date: _____

Operation: _____

Estimate – justify your thinking:

Represent - with a sketch or drawing:

Explain your sketch:

Calculate:

Write a Real Life Example or Word Problem: _____

_____

Reflect:

Communicating & Representing	Understanding & Solving	Connecting & Reflecting	Reasoning & Analyzing
1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4
Entire assessment	Draw and Calculate	Real-life & reflection	Estimate

SNAP

Number Sense

Name: _____

Date: _____

Draw to represent the value of the number:

Write to describe your picture:

Count **backwards** by _____ from the number.

Write the number in expanded form:

Create 3 equations that equal the number:

Write a real life example that shows the value of the number:

Count **forwards** by _____ from the number.

Show where the number belongs on the number line.

Reflect:

Operations SNAP

Name: _____

Date: _____

Operation: _____

Estimate – justify your thinking:

Represent - with a sketch or drawing:

Explain your sketch:

Calculate:

Write a Real Life Example or Word Problem: _____

Reflect:

Reasoning & Analyzing

Communicating & Representing

Understanding & Solving

Reasoning & Analyzing

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Number Sense Rubric
SNAP (Student Numeracy Assessment & Practice)

Competency	Emerging *The student demonstrates an initial understanding of the concepts and competencies relevant to the expected learning.*	Developing The student demonstrates a partial understanding of the concepts and competencies relevant to the expected learning.	Proficient *The student demonstrates a complete understanding* of the concepts and competencies relevant to the expected learning.	*Teacher notes for demonstration of understanding and applications beyond proficiency*
Communicating and Representing *Picture Box*	• Pictures do not show the value of the number • Inaccurate	• Pictures show some value in representing the number • Partially accurate	• Pictures are clearly communicated and represent the value of the number (e.g. base ten and/or symbols) • Accurate	
Describe Picture	• Description and elaboration of pictorial representation is not evident • Communication is not clear	• Partial accuracy in describing and elaborating on pictorial representation **AND/OR** partially communicated	• Accurately describes and elaborates on pictorial representation (e.g. legend, key, or words) • Clearly communicated	
Expanded Form	• Emergent understanding of the value of digits in their place values	• Partially accurate in demonstrating the value of each digit (40000 +2000+139=42139 OR 40000+100 +30+9=42139)	• Accurately demonstrates the value of each digit (e.g. 500+20+4 or five hundreds, 2 tens, and 4 ones)	
Understanding and Solving *3 Equations*	• Emergent use of operations	• Accurately uses grade appropriate operations in one or two equations	• Accurately uses grade appropriate operations in all three equations (see Exemplars for examples)	
Connecting and Reflecting *Real Life Connection*	• A real-life example is not provided or is not connection to the number	• A partial connection to a real-life example is provided (e.g. "I bought a house for $319")	• Connection to a real-life example is provided • Demonstrates understanding of the number value (e.g. 5347 leaves on a small tree shows understanding; "I live at 5347 Elm St," does not)	
Reflection	• With support, student is not yet able to reflect on their learning	• Can partially identify strengths and stretches (e.g. *"Everything was easy. Nothing was hard.")*	• With sentence frames and structure, can proficiently reflect on their learning (e.g."I feel confident with ___; ___ was challenging; my goal is___")	
Reasoning and Analyzing *Number Line*	• Emergent understanding of the placement of the number on a number line	• Partially correct estimate of placement of number on provided number line; benchmarks may be missing	• Correct estimate of placement of number on provided number line with at least three benchmarks and appropriate endpoints.	
Counting Forwards and Backwards	• Emergent understanding of place value, number sense, and/or skip counting	• Partially complete and accurate	• Complete and accurate; demonstrates understanding but may include a minor recording error	

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Operations Rubric
SNAP (Student Numeracy Assessment & Practice)

Competency	Emerging *Student demonstrates an initial understanding of the concepts and competencies relevant to the expected learning*	Developing *Student demonstrates a partial understanding of the concepts and competencies relevant to the expected learning*	Proficient *Student demonstrates a complete understanding of the concepts and competencies relevant to the expected learning*	Extending *Student demonstrates an insightful understanding of the concepts and competencies relevant to the expected learning*
Communicating and Representing *Entire Assessment*	• Communication (written, pictorial or symbolic) of understanding is emerging	• Communicates (written, pictorial or symbolic) partial understanding	• Communicates (written, pictorial or symbolic) clear understanding	• Communicates (written, pictorial or symbolic) insightful understanding in multiple ways
Understanding and Solving *Draw & Calculate Boxes*	• Emergent use of strategies to solve the problem and show understanding	• Strategies chosen do not lead to an accurate solution • Reasoning to solve the problem is absent	• Uses ***grade appropriate*** strategies to correctly solve the problem and show understanding	• Uses multiple strategies and/or insightful reasoning to correctly solve the problem and show understanding
Connecting and Reflecting *Real Life Example/ Word Problem*	• Emerging ability to connect mathematical concepts to real life examples	• Real life example and connections to mathematical concepts are partially developed	• Real life example and connections to mathematical concepts are evident • The example shows a clear connection to the operation	• Real life example and connections to mathematical concepts are insightful
Reflection	• With support, student is not yet able to reflect on their learning	• Can partially identify strengths and stretches "*Everything was easy; nothing was hard'*	• With sentence frames and structure, can proficiently reflect on their learning " *I feel confident with ___; ___ was challenging; my goal is___ "*	• Insightful reflection on mathematical thinking is evident
Reasoning and Analyzing *Estimate & Justify Box*	• Emerging ability to use Estimation/mental math strategies • Estimate is not yet reasonable and justification not provided	• Calculates rather than estimates *"I think it is 366 because 3x122=366"* • Strategy use is not justified *"My guess is 300 because I used mental math"*	• Reasonable estimation provided • Clearly explains strategy *"I think it is about 360 because I did 3x100=300 and 3x20=60 and added 300+60"*	• Reasonable estimation provided and insightfully explains the strategy *"I think it is about 360 because I did 3x100=300 and 3x20=60 and added 300+60 but the solution is greater than that because I rounded down""*

Grade 6/7

First Steps in Math Curated

Diagnostic Assessments & Learning Activities

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First Steps in Math

Curated Diagnostic Assessments and Learning Activities for Mathematics

In this section of the SD71 Number Sense Assessment Package can be used to support learning for students who have fallen into the emerging/developing categories from the SNAP assessment. Conversely, you will gain an even deeper understanding of all your students mathematical understanding if they all participate in the First Steps in Math Diagnostic Activities, and all students will benefit from the First Steps in Math Learning Activities.

The First Steps in Math Assessment section is organized into two sections:

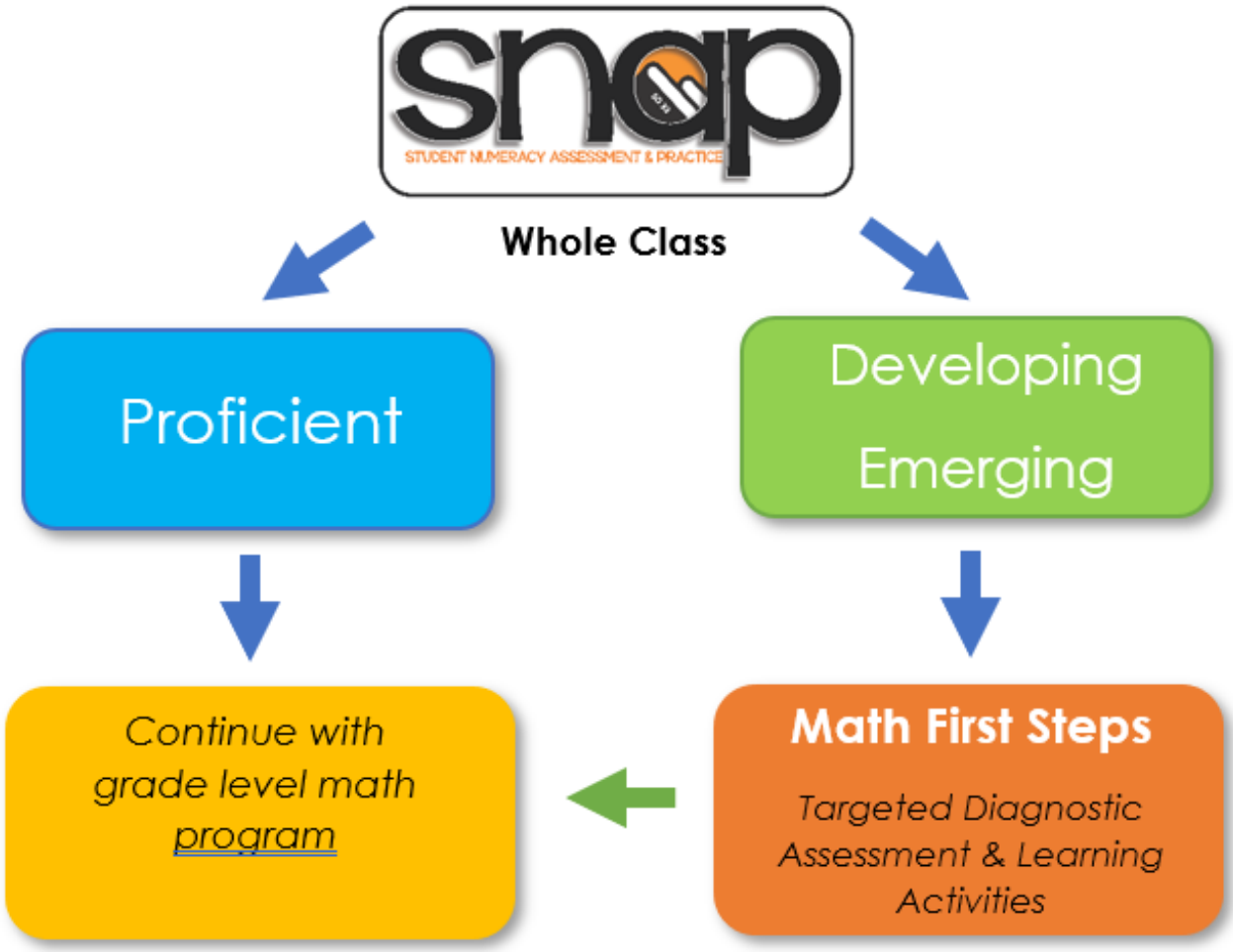
1. Diagnostic Assessments that hone in on Key Understandings of Mathematical development that students might be missing, and:

2. Learning Activities that will support learning in the key understanding areas of mathematics learning to building a strong foundation of Number Sense and relationships.

Both the Diagnostic Assessments and Learning Activities can be done one-on-one, in small groups, or whole class. Please note, in the chart on the follow page, curated grade specific diagnostic and learning activities have been hand selected to help you with your planning. If you find that there are other activities from the list that would be a better fit for your learners, please use your professional decision making to make that choice. All of the Learning Activities within a section focus on the same targeted Key Understanding, starting with the least complex to more complex. Again, all students can benefit from these activities, and they can become key components to your math program.

It is recommended that one Diagnostic and corresponding Learning Activities are focused one at a time, working with your students through those foundational key understandings at a manageable pace, revisiting multiple times. These diagnostics and activities can be woven into your existing math program and revisited every 2-3 weeks as you see fit.

All Diagnostics Assessments and Learning Activities have been curated from a larger and more comprehensive Frist Steps in Math resource that is accompanied by a 16-hour workshop to orientate and take a deeper dive into the essence of First Steps in Mathematics. If you are interested in learning more about First Steps in Math, please reach out to your School Districts Lead Teacher for Numeracy, your schools CST, or your principal.



Grade 6/7

First Steps in Math

Diagnostic Assessments

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First Steps in Math

Grade 6/7 -Curated Diagnostics & Learning Activities:

NOTE: *all the following assessments address gaps from previous grades.*

Diagnostic Assessment	Emerging Activity (EM) Suggestion	PAGE	Developing Activity (DEV) Suggestion	PAGE
Read, Write & Say Numbers (NS-Key Understanding 5)	*"Wipeout"* *"Counting in Hundreds"*	*p. 51* *p. 52*	*"Million Square"* *"Words into Symbols"*	*p. 54* *p. 54*
Fraction Task 22: "Number Lines" (FR-Key Understanding 5)	*"What Number Am I?"* *"Fraction Number Line"*	*p. 62* *p. 63*	*"Places on a Number Line"* *"Estimating"*	*p. 65* *p. 66*
If successful, then:				
Task 29: Making Lemonade" (FR-Key Understanding 7)	*"Making Juice"* *"Proportional Quantities"*	*p. 69* *p. 64*	*"Units of Measurement"* *"Ratios"*	*p. 66* *p. 67*
Decimals A-H (NS-Key Understanding 7)	*"Counting by Decimals"* *"Decimal Number Line"*	*p. 57* *p. 57*	*"Place Invaders"* *"Ordering Measurements"*	*p. 57* *p. 58*

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Diagnostic Assessment	Emerging Activity (EM) Suggestion	PAGE	Developing Activity (DEV) Suggestion	PAGE
BASIC FACTS: (COMP-Key Understanding 1)				
Interview Assessment To check for previous grade gaps. **Find the Solution: Sets A (+/-) and B (x/÷)** (COMP-Key Understanding 1)	*"Multiplication Facts"* *"Doubles and Halves"*	p. 49 p. 49	*"Multiplication Doubles"* *"Constant Calculations"*	p. 50 p. 50
Finding Factors, especially last question (OP-Key Understanding 5)	*"Constructing Arrays* *"Unknown Number"*	*p. 56* *p. 56*	*"More Arrays"* *"Factors"*	*p. 57* *p. 58*
If struggling, do: **BASIC FACTS assessment: Multiplication and Division assessment** (COMP-Key Understanding 1)	*"Multiplication Facts"* *"Doubles and Halves"*	*p. 49* *p. 49*	*"Multiplication Doubles* *"Constant Calculations"*	*p. 50* *p. 50*

Diagnostic TASK

FOCUS
Understand Numbers
- Key Understanding 5

Read, Write and Say Whole Numbers Years/Grades 3–7

Purpose

To explore the limits of children's writing of large numbers and to expose their personal rules or misconceptions when writing such numbers.

Producing work samples

Whole class or small group observations

Provide each student with copies of the 'Read, Write and Say' worksheet. Call out the following numbers for children to write for questions 1–6.

1. Sixty three
2. One thousand twenty
3. Twenty six thousand fifteen
4. Five hundred six thousand fifteen
5. One million five
6. Five billion, thirty six million, four hundred seven thousand four.

Children complete the rest of the sheet independently.

If needed, interview individuals and ask them to explain how they knew to write the number in the way that they did. The purpose of this is to uncover any invented rules that children may be using.

Read, Write and Say Numbers

Name _____ Year/Grade _____ Date _____

Instructions: Write the numbers the teacher says. Here is an example.
If the teacher says ***nineteen*** you write ***19***.

1. _____ **4.** _____

2. _____ **5.** _____

3. _____ **6.** _____

Write these numbers in words:

504

1,768

250,000

13,648

6,003

13,806,009

Diagnostic TASK

FOCUS
Understand Fractions
- Key Understanding 7

Making Lemonade

Grades 4–8

Purpose

To show whether students are able to use fractions as ratios to work out a larger amount of liquid from a smaller one.

Interpreting Students' Response

Some students who have had little experience with ration, and therefore have little knowledge of it, will struggle to find a strategy to work out the answer to this problem. Other students who have had little experience with ratio may be able to use their existing knowledge of fractions to work with the numbers in some way, although they will struggle to find a sensible answer.

Some students who have had some experience to the idea of rations will be able to work out that $\frac{1}{5}$ of 10 liters is 2 liters and $\frac{4}{5}$ of 10 liters is 8 liters. They may do this first by working out how much lemonade juice and how much water are needed for the liter and then multiplying these to find out how much for 10 liters.

Students who have a good understanding of ratio should be able to choose to use the written algorithm for multiplication fractions - $\frac{1}{5}$ x 10 and $\frac{4}{5}$ x 10 – to arrive at the correct answers and should be able to explain why they have chosen to multiply.

Making Lemonade

Name _____ Grade _____ Date _____

Some children were making lemonade for the school fundraiser. They made 1 liter of lemonade using $\frac{1}{5}$ lemon juice to $\frac{4}{5}$ water and thought that this tasted just right. However, they need to make 10 liters for the fundraiser.

How much lemonade juice and how much water would they need to use to make it taste the same?

Lemon juice _____ Water _____

Use the space below to show how you worked it out.

Diagnostic TASK

FOCUS
Understand Numbers
- Key Understanding 7

Decimals

Grades 5-9

Purpose

To see the extent of students' understanding of decimals and to uncover preconceptions and misconceptions

Materials

- Line Masters: Decimals (Sets A–H)

Instructions

Have students complete the questions individually. You may need to conduct some individual interviews where students' reasoning is not clear from the written explanation.

Note: Students usually find some questions easier than others. Therefore, you may choose to break up the worksheets and present sections one at a time.

Name _____ Grade _____ Date _____

Line Master **Decimals (Sets A–D)**

A Compare and Order

Circle the biggest number in each group of three.

i) 5 436 or 547 or 56

ii) 6.78 or 45.6 or 345

iii) 3.521 or 3.6 or 3.75

iv) 15.4 or 15.56 or 15.327

v) 4.09 or 4.7 or 4.008

Swan, M. (1983)

B Money

i) When James used his calculator to see how much his shopping came to it showed 14.5.
How much is that in dollars and cents?

_____

ii) Rachel purchased 4 balls.
She worked out the price for one ball on the calculator.
The result was 6.125. How much is that in dollars and cents?

_____

C Renaming Decimals as Fractions

Write these numbers as fractions.

i) 0.67 _____

ii) 0.341 _____

iii) 0.2 _____

D Naming Digits after the Decimal Point

i) What does the 3 mean in 0.236?

_____

ii) What does the 2 mean in 0.236?

_____

iii) What does the 6 mean in 0.236?

_____

Name _____ Grade _____ Date _____

Line Master **Decimals (Sets E–F)**

E Ordering

Sonya said, "When we put books on the library shelf we put 65.6 before 65.125 because 6 is less than 125," but Tao didn't agree.

Who is right? _____

Why do you think that?

Write your explanation in this box.

F Counting On and Back by Decimal Numbers

Write down the next 2 numbers in each sequence.

a) 0.2, 0.4, 0.6, _____, _____
(add 0.2 each time)

b) 0.3, 0.6, 0.9, _____, _____
(add 0.3 each time)

c) 0.92, 0.94, 0.96, 0.98,
_____, _____
(add 0.02 each time)

d) 1.13, 1.12, 1.11,
_____, _____
(take away 0.01 each time)

Name _____ Grade _____ Date _____

Line Master **Decimals (Sets G–H)**

G Quantity

Paper clips come in boxes of 1000. Abi counted the loose paper clips in a tray and said there were 1260. Jeremy said, “That's 1. 26 boxes of paper clips.”
Could they both be right?

Yes ☐ No ☐

Why do you think that?

H Number Sequence

How would you use a calculator to generate this number sequence?

2.0, 0.2, 0.02, 0.002

Diagnostic TASK

FOCUS
Understand Computations
- Key Understanding 1-6

Find the Solutions

Sets A and B suitable for Grades 3-4
Sets B and C suitable for grades 5-7

Purpose

To see what strategies students use to solve problems requiring addition, subtraction, multiplication, or division

Materials

- Line Master: Find the Solutions (Set A)
- Line Master: Find the Solutions (Set B)
- Line Master: Find the Solutions (Set C)

Instructions

1. Provide each student with the appropriate line master(s). Read the questions with them to make sure they understand what they need to find out.

2. Explain to the students that they need only work on the problems that they think they can do. It is important to end the interview when the child indicates the questions are getting too hard. You may wish to copy only the left-hand set of questions for the students you believe might feel inadequate for not progressing to the end.

Individual Interview: Sit with a student as she/he works out each question. When she/he has an answer, ask about the strategy the student used. The focus is on revealing some of the repertoire the student has developed; commenting on whether the answer is right or wrong is inappropriate in this situation. It is important to end the interview when the student indicates that the questions are getting too hard. Record the numbers the students uses as she/he partitions, re-arranges, orders, and operates on a separate copy.

Whole-Class Task: Provide each student in the class with a copy of the questions. Ask them to read the question, and work out the answer in their head. Interview a few students of different abilities to find out how they thought of the numbers and how they worked each question out. Record the numbers they use as they partition, re-arrange, order, and operate on a separate copy.

Name _____ Grade _____ Date _____

Line Master **Find the Solutions (Set A)**

1. On the bus there are 25 children from Mr. Foster's class and 30 children from Mr. Singh's class. How many children are on the bus?	2. There were 100 paper clips in the box. We have used 37 of them. How many are left?
3. Your mother made 24 pancakes in the first batch and 18 in the second batch. How many pancakes did she make?	4. Sean's family is on the way to town. They have already travelled 15 km and the town is 65 km from their home. How far do they still need to travel to reach town?
5. There are 18 slices of bread in a loaf. How many slices will there be in 5 loaves?	6 There was $120 in $10 bills. How many bills should there be?

Name _____ Grade _____ Date _____

Line Master **Find the Solutions (Set B)**

1. In Joe's school each class has 25 children in it. The school has 16 classes. How many children are in the school?	2. Crystal has 375 newspapers to deliver. She has delivered 127. How many does she still have to deliver?
3. Every week Ted earns $235. Does he earn more or less than $900 every 4 weeks? How do you know?	4. Jeremy has to deliver 226 newspapers. How many more does he need to deliver until all of the 537 newspapers in his paper route are delivered?
5. Abi has two short paper routes. She delivers 374 in one route and 227 in the other. How many newspapers does she deliver altogether?	6. There were 1035 newspapers to deliver and 10 delivery people. How many papers did they each deliver?

Name _____ Grade _____ Date _____

Line Master **Find the Solutions (Set C)**

25 + 30	25 x 16
24 + 18	375 – 124
18 x 5	226 + ___= 537
100 – 3	374 + 227
15 + ___ = 65	1035 split into groups of 10
120 split into groups of 10	27 x 16
235 x 4 Estimate. Will the answer be more or less than 900? Why?	

Diagnostic TASK

FOCUS
Understand Operations
- Key Understanding 5

Finding Factors

Years/Grades 5–7

Purpose

To find out if children understand and can use the inverse relationship between multiplication and division.

Equipment

Worksheet and a calculator for each child

Producing work samples

Individual interview or whole class activity

The question in the box at the bottom of the page is the crucial part of this task and will give you the most significant information about what students know.

It will be necessary to remind students what a factor is and let them practise with easier numbers like 12 or 15. The students could be given some factor activities on the board, which are similar to the first two boxes to enable them to become familiar with the idea of factors.

This task can be used as an individual interview or as a whole class activity. It may be beneficial to give the top part of the task to the whole class but withhold the last box from the sheet and use it in an individual interview.

Finding Factors

Name _____ Year/Grade _____ Date _____

Find factors for these numbers

81 _____ _____ _____ _____ _____ _____ _____ _____ _____

Which numbers did you try? _____

Which ones were hardest to find? _____

How did you work it out?_____

_____

105 _____ _____ _____ _____ _____ _____ _____ _____ _____

Which numbers did you try? _____

Which ones were hardest to find? _____

How did you work it out?_____

_____

Sam wondered if 13 was a factor of 105 but did not know what to put into the calculator to find out. Explain to Sam what he could do to find out.

Grade 6/7

First Steps in Math

Learning Activities

First Steps in Math

Grade 6/7 -Curated Learning Activities & Materials:

NOTE: *all the following assessments address gaps from previous grades.*

Emerging Activity (EM) Suggestion	Materials	PAGE	Developing Activity (DEV) Suggestion	Materials	PAGE
"Wipeout" *"Counting in Hundreds"*	• Calculators	p. 51 p. 52	*"Million Square"* *"Words into Symbols"*	• *cm paper coloured* • *See Appendix -Line Masters 2* • *Scissors* • *Glue* • *Newspapers/magazines*	p. 54 p. 54
"What Number Am I?" *"Fraction Number Line"*	• Paper strips/register tape	p. 62 p. 63	*"Places on a Number Line"* *"Estimating"*	• *Fraction cards* • *Paper Strips*	p. 65 p. 66
"Making Juice" *"Proportional Quantities"*	• *Linking cubes -2 colours*	p. 69 p. 64	*"Units of Measurement"* *"Ratios"*	• Paper strips/register tape	p. 66 p. 67
"Counting by Decimals" *"Decimal Number Line"*	• *Calculators* • *Clothes line/string* • *Clothes pegs*	p. 57 p. 57	*"Place Invaders"* *"Ordering Measurements"*		p. 57 p. 58
"Multiplication Facts" *"Doubles and Halves"*		p. 49 p. 49	*"Multiplication Doubles"* *"Constant Calculations"*	• *Calculators*	p. 57 p. 58

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"Constructing Arrays *"Unknown Number"*	• *Colour paper-multiple colours* • *Scissors*	p. 56 p. 56	*"More Arrays"* *"Factors"*	• *Counters/cubes* • *Calculators*	p. 57 p. 58
"Multiplication Facts" *"Doubles and Halves"*		p. 49 p. 49	*"Multiplication Doubles* *"Constant Calculations"*		p. 50 p. 50

first steps

First Steps in Mathematics

Number Sense

Whole and Decimal Numbers, and Fractions

Improving the mathematics outcomes of students

PEARSON

Government of Western Australia
Department of Education and Training

FSIM006 | First Steps in Mathematics: Number Sense

First Steps in Mathematics: Number
Number Sense: Whole and Decimal Numbers, and Fractions

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Sample Learning Activities

Grades 3-5: ★ ★ ★ Major Focus

Animal Patterns

Have students create an animal using Pattern Blocks, and then say how many blocks were used. For example: *My cow was made with eight triangles.* Ask: How many triangles would you need for five cows? Ask students who did not use materials to solve this and share with the class how they worked it out.

Same Numbers

Ask students to use their own strategies to solve and record solutions to multiplication problems involving the same numbers. For example: Jeremy can only carry seven plastic milk containers at a time to the recycling bin. How many does he take in four trips? Ask students to share the number sentences they used, say why they are the same and why the answers are the same.

Multiplication Facts

Help students to build up sets of related multiplication facts. For example: Ask students to draw one tricycle and say how many wheels then two tricycles and how many wheels, and so on. Encourage students to look for the pattern and say why five tricycles must have 15 wheels. Have students record the number sentences as the pictures are drawn to list the first five or six multiples of three.

Today's Number

Write a number on the board. Ask students to suggest calculations with that number as an answer. Record their calculations on the board. Ask: Are there any number sentences that belong together? Why? As students mention the four operations, build different groups. Ask: Can we rearrange the number sentences so they are in order? How can each set be extended?

Forgotten Facts

Ask students to explain to a partner how they could work out a fact they do not know or have forgotten. For example: *To find 6 x 5, I know it's 5 x 5 and another five.*

Doubles and Halves

Have students use doubles and halves to multiply and divide. For example: 4 x 7 is double double seven, which is double 14, or 28; 24 ÷ 4 is half of half of 24, which is half of 12, or six.

Multiplication Doubles

Have students relate known multiplication "doubles" to the harder multiplication facts. For example: As a way of remembering 6 x 8, students could make a 6 x6 grid and build on until they make 6 x 8. They work out tables they could put together to help work out the original, such as 6 x 6 and 2 x 6. Ask: Could you use addition instead of the 2 x 6? Which is easier?

Extending Doubles and Halves

Extend students' use of doubles and halves to find answers to multiplication, such as 8 x 12. For example: Halve eight and double 12 gives 4 x 24; halve four and double 24 gives 2 x 48; halve two and double 48 gives 1 x 96.

Concentration

Have students use addition and multiplication examples to construct cards to play games such as Concentration. Pairs of cards are made by putting together different representations of the same number. For example: 3 x 2 and 2 + 2 +2 would be a matching pair.

Constant Calculations

Have students use the constant function on the calculator to find multiples. Forexample: When learning the four times table, press 0 + 4 then = = = to find the multiples of four. Ask students to predict what will be next and then to verify their prediction. Ask: Why can't a number with seven in the ones column be a multiple of four?

Grid Patterns

Have students make a multiplication grid by placing numbers along the top and down the side and the answers within the grid. (See right.) Look for patterns within the grid. Ask: Why are the numbers above the diagonal the same as below? How can this help find answers to tables you don't know?

Looking for Patterns

Have students investigate patterns in the answers of times tables. For example: In the nine times table, the digits of each answer add to nine, the numbers in the ones column go up by one and the numbers in the tens column go down by one.

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Sample Learning Activities

Grades 3-5: ★ ★ ★ Major Focus

Patterns in Numbers

Ask students to make their own 100-chart, arranging the numbers in whatever number of rows and columns they like. Have students use their chart to look for patterns in the numbers. Then, ask students to quickly find a number, such as 67 or 42. Show students a 10 x 10 100-chart (See Appendix: Line Master 5) and ask: What changes from one row to the next? Why? What changes in the other charts? Why? In which chart is it easier to find particular numbers? Why? Have all students make a chart for their personal use. Encourage students to extend their chart over time.

1	13	25
2	14	26
3	15	27
4	16	
5	17	
6	18	
7	19	
8	20	
9	21	
10	22	
11	23	
12	24	

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21			

Number Cube Rolls

Ask pairs of students to take turns to throw a number cube and record results in a row on squared paper, which is 5 squares wide. Have students choose which square to enter each digit in order to make the largest possible number. When both students have made a five-digit number, the player with the largest number chooses a different rule, such as *Make the lowest number or the number closest to 50 000*.

Wipeout

Play with the whole class. Enter a number, such as 256, into a calculator. Ask: How can we make the 5 a 0? (*Subtract 50*.) Why did you do that? What number have we got now? Eliminate the 2. Try larger numbers when students are ready. Later, have students play Wipeout in pairs, taking turns to give each other instructions. Encourage students to try larger numbers, such as 946 256.

10 Times as Great

Organize students into pairs. Invite students to use their calculators to find out what numbers are 10 times as great as the given numbers, such as 30, 172, 109, 200, 210, 4550. Say: Can you see a pattern? Try to explain to your partner why that happens. What will 10 times 7568 be? Test it and see.

Counting in Hundreds

Ask students to use constant addition on a calculator to count in hundreds. Have them predict which number will come next, then press = to verify. Ask: How many hundreds did you put in to make 900? How many hundreds are in 1000 (2000)?

Multiplying by 10

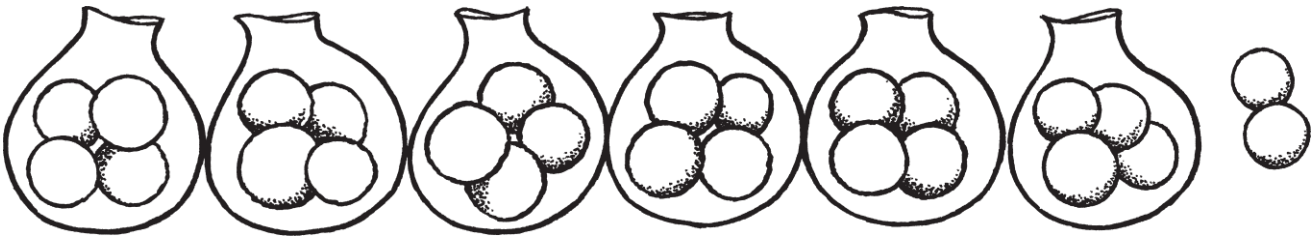
Have students predict the effect of multiplying a number by 10. Use the overhead projector calculator and begin with any single digit. Ask: If we multiply this number by 10, what will the number be? If we multiply by 10 again, what will the number be? How many tens in 100 (1000)?

Three-Digit Numbers

Ask students to use grid paper to draw a diagram that shows the size of each of the digits in a three-digit number, such as 888. Ask: How do you know you have the size right for each of the digits? How many times as big is the second 8 than the first? Later, have students represent the size of the digits in other three-digit numbers, such as 256, without using grid paper.

Marbles

Have students explain the meaning of the digits in a numeral using materials that are deliberately not grouped in standard ways—that is, not in tens—such as 26 marbles. For example, students put out 6 bags of 4 marbles and 2 more marbles. Ask: How many marbles? Have students write down how many. Record the correct answer on the board. Point to one digit and ask students to show their partner the number of marbles it refers to. Point to another digit and repeat. Repeat this activity with other collections that are not grouped in tens, for example, 3 bundles of 10 Popsicle sticks and 13 singles.



Sample Learning Activities

Grades 5-8: ★ ★ ★ Major Focus

800 Game

Have students investigate the multiplicative relationship between places. Organize students into pairs. Then, give each student a card labelled "8" and up to five cards labelled "0". Ask each student to make a different number with the digit cards. For example, the first student could make 8; the second student could make 800. Ask: What number sentence would you key into your calculator to change your number so that it is the same as your partner's? Have students share their number sentences, then ask: Who used addition and subtraction? Who used multiplication and division? Refer to a chart that shows the cyclical pattern of the number system (See Appendix: Line Master 7) to emphasize how multiplication and division match the relationship between the places. For example, say: To make 8 into 800, you can key 8 x 10 x 10 or 8 x 100 into your calculator. To make 800 into 8, you can key 800 ÷ 10 ÷ 10 or 800 ÷ 100 into your calculator. Have students repeat the activity making a different number with their cards and then use the chart to explain why the number sentence they chose actually works. Later, extend the activity to include a decimal point and more zeros.

Counting Crowds

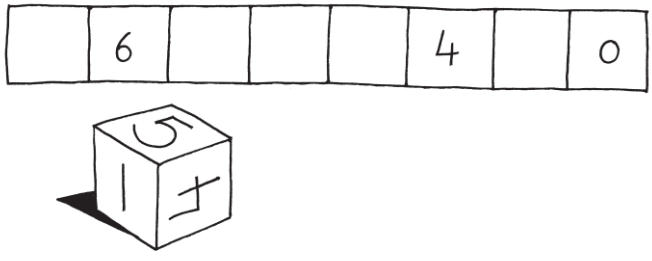
Have students solve problems such as: The number counter at the entrance to the fair reads 9999 (10 999, 99 999) after the person in front of you goes in. What will the counter read after you go in?

One-tenth As Much

Ask students to use their calculators to find out what is one-tenth of each of these numbers: 30, 172, 109, 200, 210, 4550. Have them record their answers, then ask: What did you do? Did you need to do the same for each number? Repeat the activity with decimal numbers such as: 3, 2.1, 1.72, 1.09, 45.5.

Number Cube Rolls

Have students make either the largest or the smallest number possible from a fixed number of number cube rolls. They can use up to eight squares in a row to record a digit from each roll of the cube. Each student has a "free zero", which he or she can place anywhere in the row. After a few rounds, ask: What do you need to do to make the largest (smallest) number possible? Why?



Million Square

Help students to create an area of one million square millimetres. Draw out the relationships between the powers of ten and successive places. Use 1-mm grid paper (See Appendix: Line Master 2) and draw around 1 square millimetre, then 10, 100, 1000, 10 000 square millimetres and label them. Combine cut-outs of 10 to 1000 square millimetres to create a million square. Ask: How much space do you think we'll need on the bulletin board for this? (See Case Study 4, page 94.)

Changing Places

Ask students to use materials, such as Base Ten Blocks, to model the relationship between places. To begin, show students the smallest Base Ten cube. Ask: What number is this? (*1*) Then, show the next largest Base Ten cube, and ask: What number is this? (*1000*) How many times as big is this than the first cube? (*1000 times as big*) What do you think the next-sized cube will look like? Do you have enough large blocks in the school to build the next-sized cube? Do you have enough to build just the frame of the cube? Have students write the numbers for each cube. Say: Imagine what the fourth cube looks like. How do you say it?

Words into Symbols

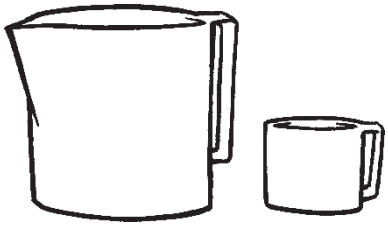
Have students rewrite large numbers written as words into symbols. Ask students to show all of the places. A good source for large numbers is newspapers. For example, "The budget deficit is 8 billion."

Sample Learning Activities

Grades 3-5: ★ ★ ★ Major Focus

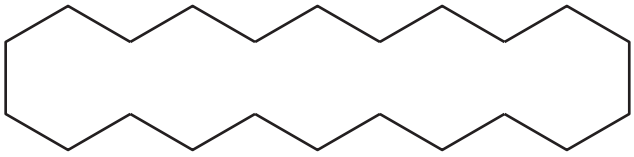
Jugs

Organize students into groups, and have each group work out how many little cups it takes to fill a jug. They then take a number (say, seven) of identical jugsand fill them. Ask: Is it possible to say how many cupfuls that would be? Model the process with students by asking: Do you need to measure the cups in each jug? How many cups are there in each jug (say, four cups)? So there are four cups in each jug, and how many times did we take four cups (seven times)? So we took four cups, seven times; we took seven groups of four cups. Have students draw a picture and record: *Seven groups of four cups. 7 times 4. 7 x 4.*



2-D Shapes

After completing *Jugs*, above, have students say how many hexagons will cover a Pattern Block shape (an outline of a shape produced by tracing around five hexagon shapes placed together), then how many trapezoids fit on the hexagon. Use this to say how many trapezoids will fit on the original shape. Focus on the five times two, rather than counting by 2s. Repeat to find out how many blue rhombuses (five times three) and how many triangles (five times six) will cover the same shape.



More or Less than One

Pose the following problem: Six students divide 18 bars of chocolate between themselves. How much will each one get? More or less than one bar? Do you need to work out how many each gets in order to answer the question? What if some students leave early and do not share the chocolate bars, so there areonly four left to share the 18 bars? Will those remaining get more or less now? What if the four students had only six chocolate bars to divide between them? Would they get more or less than one each? What if four students had three chocolate bars to share? Discourage students from doing the calculation inorder to decide.

Constructing Arrays

Have students construct arrays for a given number using grid paper, pegboards or blocks. Ask them to record each array using two multiplication sentences, such as 6 x 4 = 24, 4 x 6 = 24, and two division sentences, such as 24 ÷ 6 = 4 and 24 ÷ 4 = 6. Ask: What do each of the numbers show in the array? How is 6 x 4= 24 the same as 4 x 6 = 24? How is it different? Can we say 24 ÷ 6 = 4 and 24÷ 4 = 6 are the same? Why not?

Unknown Number

Ask students to write open multiplication sentences to solve problems, and then say whether the unknown number is the number of groups or the amount in each group. For example: Tanya has five times as many marbles as Jill; if Tanya has 70, how many does Jill have? Or: Jill has five marbles; how many times as many marbles does Tanya have if she has 70? This could be written either as 5 x □ = 70 or as □ x 5 = 70. Ask students to read their number sentence to others and say how it represents the situation. Ask: What does the missing number represent in each case?

Fraction Problems

Have students use materials, such as paper or diagrams, to solve multiplication problems involving fractions. For example: If 32 students need half a piece of paper each, how many whole pieces of paper will be needed? Ask students to reflect on what they did and record this by writing a description of their thinking. Ask: Which sign can be used to show how you found your answer? How can you decide between multiplication and division? For example: Is it $\frac{1}{2}$ x 32 = 16 or 32 ÷ 2 = 16, or can it be both?

Multiplication and Division Stories

Have students review a collection of different multiplication and division stories (See Background Notes, page 25) and decide whether the unknown number represents the whole, the number of groups or the quantity in each group. Ask: If the whole is the unknown, which operation can be used to work it out? If the number of groups or the quantity in each group is unknown, which operation can be used? Why?

Halving Quantities

Students extend *More or Less*, on page 75, by using the calculator to work out the results of halving quantities. Ask: What keys can you use to find half of six? How can you multiply by a half using the calculator (multiply by 0.5)? How is 6 ÷ 2 the same as 0.5 x 6?

Sample Learning Activities

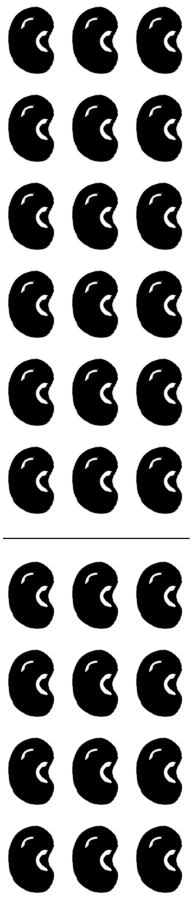
Grades 5-8: ★ ★ ★ **Major Focus**

Small Units

Extend *Jugs*, on page 76. Have students create groups to count large collections or measure quantities with small units. Present situations where students have to say how many dots there are in a large array, how wide the classroom is in centimetres, or how many eyedroppers of water there are in a jug. Ask students to decide how they could efficiently work out the amount without counting the single units by ones or repeatedly counting the number of units in each group. How would knowing the number in a row and the number of rows in the array help? How would knowing that a metre ruler is the same as 100 cm help? Encourage students to use multiplication to find out how many or how much.

Multiplication and Division

Encourage students to recognize the relationship between multiplication and division. Ask them to draw a 7 x 8 array, and then record multiplication and division number sentences about the array. Ensure that a range of examples are drawn out, such as 7 x 8, 8 x 7, 56 ÷ 8, 56 ÷ 7, $\frac{1}{7}$ x 56, $\frac{1}{8}$ x 56.



10 x 3 = (6 x 3) + (4 x 3)

Arrays

Have students use materials, such as beans or cubes, to construct arrays then partition them in different ways and record a number sentence for each. For the 10 x 3 array shown left, they might write: *10 x 3 = (6 x 3) + (4 x 3) or 30 = (6 x 3) + (4 x 3)*. Ask: How can you check that your partner's number sentences match the array your partner has made without calculating the answer? Which numbers did you use—the number in each row or the number of rows?

More Arrays

Extend *Arrays*, above, by posing problems such as: If you had four rows of three blocks and added some more rows to make 11 rows altogether, how many rows did you add? How many blocks did you add? If you had some rows of five and added four more rows of five to make 12 rows of five, how many rows did you start with? How many blocks did you start with?

Which Operation?

When solving the following problems, have students say what is missing—the whole quantity, the number of groups or the number in each group—to justify their choice of operation:

- The local hockey arena can hold 525 people and has 15 rows of seats. How many seats are there in each row?
- How many students can have 6 markers if there are 55 in the container?

- If 28 students drink about $\frac{3}{4}$ of a litre of juice each, how much juice will have to be ordered for the party?

Ask: What operation is required when you know the number of groups and the number in each group, but not the whole? What operation(s) can be used when you know the whole amount and the number of groups, but not how many in each group? What operation(s) can be used when you know the whole amount and the number in each group but not how many groups? Why can some situations be represented with both division and multiplication?

Division Questions

Have students write division questions from multiplication problems. Begin with a multiplication situation such as: Shane's car travelled three times as far as Jenny's. Jenny's car travelled 2 m, so Shane's car travelled 6 m. Ask students to construct two division questions from this multiplication situation. What questions could you ask if you know the whole amount (Shane's distance), but don't know the number in each group (Jenny's distance)? What question could you ask if you know the whole amount, but don't know the number of groups (how many times more)?

Equivalent Operations

Encourage students to use equivalent number sentences to make calculating easier. For example: Isabella needed to work out 56 ÷ 8. She knows her multiplication tables well but does not know her divisions as well. How could Isabella use multiplication to help her solve the division sentence? Ask students to draw an array or a diagram to show which piece of information is missing (the whole amount, the number of groups or the number in each group) and explain why they can work it out either way.

Fractions

Use pizza-sharing activities where three pizzas are shared between four people to draw out the idea that if anything is shared between three people, each person gets one-third of it. Therefore, 2 ÷ 3 is the same as $\frac{1}{3}$ of 2. Have students demonstrate with materials (paper, fruit, modelling clay), diagrams and a calculator with fraction functions, that 4 ÷ 5 is the same as $\frac{1}{5}$ of 4 and $\frac{1}{4}$ of 3 is the same as 3 ÷ 4.

Factors

Set students a target number, such as 105, and ask them to use their calculator to test whether various numbers are factors. Many will use "guess and check", that is, to test whether 13 is a factor they will guess what you would multiply 13 by to get 105 and test it with their calculator. Challenge them to find an easier way and draw out that they can use the inverse relationship between division and multiplication. For example: 105 ÷ 13 = 8.07 so 13 is not a factor; 105 ÷ 15 = 7, so 15 is a factor.

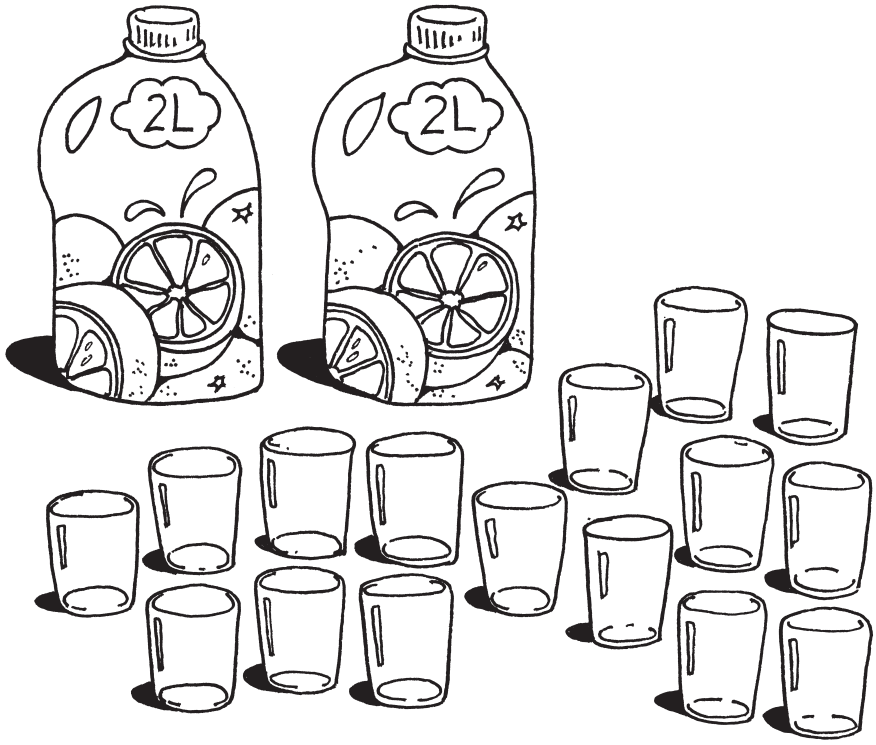
Grades 5-8: ★ ★ ★ Major Focus

Sharing Problems

Have students group to solve sharing problems. For example: Predict what each share will be if 420 paper clips are to be shared between 12 students. Ask: How many will I need to pull out for 12 students to get one paper clip each? What if I pull out another handful of 12, how many each? How many have I used up so far? How many groups of 12 will I need to pull out to share all the 420 paper clips? How many will each student have? Note that the number of groups gives the number in each share.

Rewriting Number Sentences

Pose problems with larger and more complex numbers so that students see a reason for rewriting a number sentence in a different but equivalent form. For example: Two 2-L (4000 mL in total) containers of juice were emptied into 16 large glasses. How much was in each glass? Students record the possible number sentences and say which they prefer to use and why: 16 x ☐ = 4000 or 4000 ÷ 16 = ☐.



Sample Learning Activities

Grades 5-8: ★ ★ ★ Major Focus

Counting by Decimals

Have students use the constant function on their calculators to count by 0.2. Ask them to read and list each number as it appears on the display. Stop students at 1.8 and invite them to predict what the next number will be. Have students check to verify their predictions. Ask: Why can't the next number after 1.8 be 1.10? Then, ask students to continue the count to 2.8. Repeat the predict-and-check cycle through 3.8, 4.8, and so on. Select students to say the number sequence forwards and backwards.

Decimal Number Line

Hang a clothesline across the classroom. Set it up as a decimal number line, with a card labelled "2" on the extreme left of the line and a card labelled "3" on the extreme right. Write "2.5" on another card and ask where it should go. When students answer correctly, pin the number card in position on the line. Ask students to write another number to add to the line and to explain why they have placed their card in that position. Later, extend the activity to thousandths and decimals with different numbers of places.

Place Invaders

Extend *Wipeout*, on page 68, so that numbers can only be wiped out from the ones place. Discuss with the students how they may need to multiply by 10s (if tenths are present) or 100s (if two decimal places) to remove the decimal first. For example, for the number 256.37, multiply the number by 100 to make 25637 and then subtract 7. Ask: How do you know what to multiply or divide by to get the digit into the ones place?

Number Cycles

Ask students to use an extended place-value chart that includes decimal places.

hundreds	tens	ones	hundreds	tens	ones		tenths	hundredths	thousandths
thousands			ones			•	fractions		
3	4	6	4	2	7	•	1	2	5

How Many?

Recount a story about an office that uses an average of 1.23 cartons of paper clips each month. Given that each carton has 10 boxes of 100 paper clips, have students decide how many paper clips the office uses on average. Ask: Does this seem reasonable? Point to the individual digits in 1.23 and ask how many paper clips each digit represents.

Recording Measurements

Ask students to decide what the decimal point shows when using it to record measures, such as their height, or how high and how long they can jump. Have students record each measurement in centimetres (132 cm), metres and centimetres (1 m, 32 cm), and metres (1.32 m). Ask: What does the 1 in 1.32 mean? What does the decimal point do? What does the .32 mean? Emphasize that the decimal point distinguishes metres from parts of a metre.

Decimal Fractions

Use decimetre squares of 1-mm grid paper (See Appendix: Line Master 2) as units to show how successive division by 10 relates to the places. Cut the grid paper into ten pieces, take one-tenth and write 0.1; cut that piece into ten pieces and take one-tenth, then write 0.01, and so on. Cut a square into two pieces, keeping to grid lines, and calculate the decimal fraction of each piece. Ask: If you are using a calculator to add the two numbers together, why must the result be 1?

Ordering Measurements

Invite students to order a series of measurements in metres (litres, kilograms) and say what the digits to the right of the decimal point mean. Perhaps these figures could be taken from the jumps and throws recorded at a recent track and field meet. Then, ask questions, such as: Which is longer 2.34 or 2.5? Why? How many centimetres is 2.5 metres?

Lengths as Decimals

Ask students to record lengths as decimals on a metre stick. Ask: If we need to be more accurate than measuring to the nearest 10 centimetres, how could we make smaller measures on our metre stick? Focus attention on splitting a tenth into tenths and renaming these as hundredths. Ask students what fractional part of the metre each place represents. For example, ask: What does the first place after the decimal point represent? What does the second (third) place represent?

Sample Learning Activities

Grades 3-5: ★ ★ Important Focus

What Number Am I?

Pose the following problem to students: I am less than one but more than zero. I am bigger than one-half. Have students guess the number and then discuss the strategies they used to work out the answer. Later, ask them to make up their own fraction clues to give to the class.

The Frog and the Flea

Pose the following problem to students: A frog and a flea had a jumping contest. Each of the frog's jumps was one-third of a unit long. Each of the flea's jumps was one-quarter of a unit long. The winner was the one who reached four units in the fewest jumps. Predict which creature won and explain why. Encourage students to represent the jumps on a number line to check their predictions. Then, ask: What if the race was longer?

Fraction Tapes

Help students to see how fractions fit with whole numbers. First, have them fold identical lengths of cash register tape into various fractional parts. Then, ask students to label the folds in sequence; for example, from $\frac{1}{4}$ to $\frac{3}{4}$, then label the start $\frac{"0"}{4}$ and the end $\frac{"4"}{4}$. Ask: How is the half marked on this tape different from, say, half an apple? Draw out the idea that the fractions on the tape show a position on the tape. (See also Case Study 3, page 149.)

Allowance

Pose this problem to students: Mary and John each spent a quarter of their allowance. Is it possible for Mary to have spent more money than John? What if they had spent half of their allowance? Have students justify their responses in terms of the size of the whole.

Estimating Fractions

Ask students to estimate the size of fractions of things in their environment. For example, say: Show me a third of the bulletin board (your desk, the wall). Ask: How did you decide where a third is?

Finding Fractions

After activities such as *Estimating Fractions*, above, ask students to fold a paper strip to find a given fraction. Give students different-sized strips of paper. Then, ask students to find someone else in the room with the same sized strip and compare fractions. Ask: How do you know that the fractions show the right amount? How can you be sure?

Grades 3-5: ★ ★ Important Focus

Estimating Positions

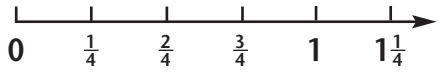
Extend *Finding Fractions*, on the previous page, by giving students several strips of paper the same size. Ask them to estimate without folding, the position of a half, a third, a quarter, three-quarters and two-thirds, each on a different strip. Then, have students place their strips together and review their decisions, making changes to the position of the fractions where appropriate.

Cheesecake

Have students think about the size of fractions to solve word problems. For example, say: Dad told Louise and Matthew that there were two pieces of cheesecake left in the fridge. One piece was $\frac{1}{3}$ of the cheesecake. The other piece was $\frac{1}{4}$ of the cheesecake. Dad said the older child should get the bigger piece. He gave Louise $\frac{1}{3}$ of the cheesecake and Matthew $\frac{1}{4}$ of the cheesecake. Who do you think is older: Matthew or Louise? Have students draw diagrams to explain their answers.

Fraction Number Line

Draw a number line on the ground or on a large sheet of paper with units and half units marked. Have students jump in units, half units and/or quarter units, counting as they go, such as *one-quarter, two-quarters, three-quarters, one, one and one-quarter*.

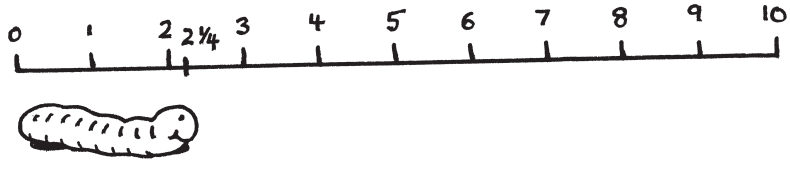


Sharing Chocolate

Pose this problem to students: Last night, I was offered the choice of half, a quarter or a third of a chocolate bar. Which one would have been given me the most chocolate? Have students use a number line to justify their responses.

Comparing Lengths

Give each student a number line marked in units from 0 to 10. Then, ask students to draw a worm 2 $\frac{1}{4}$ units long. Repeat this activity with a number of different lengths. Have students mark their worms' positions on the number line and talk about how they determined where the worm would begin and end.



Sample Learning Activities

Grades 5-8: ★ ★ ★ Major Focus

Less Than 100

Ask: What is the biggest number you can think of that is less than 100? Use a long strip of 1-mm grid paper (See Appendix: Line Master 2) to represent a number line segment between 99 and 100. Have students begin by marking $99\frac{1}{2}$ on the strip and then ask them to add numbers larger than this, such as $99\frac{3}{4}$. Ask students to indicate and justify the position of their numbers on the line.

Ordering and Comparing Fractions

Ask students to use a half, a third, a quarter and three-quarters as reference points to determine the size of a fraction, or to order and compare fraction numbers. For example, ask: Is $\frac{5}{8}$ smaller or bigger than a half? Does knowing that $\frac{4}{8}$ is a half help? Use what you know to say whether $\frac{8}{14}$ is more or less than $\frac{5}{8}$.

Have students use these strategies to order sets of fractions with unlike numerators and unlike denominators; for example: $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{9}{10}$

Fraction Cards

Have students order sets of fraction cards with:

- like denominators; for example: $\frac{3}{4}$, $\frac{1}{4}$, $\frac{2}{4}$; or
- like numerators; for example $\frac{2}{3}$, c, $\frac{2}{7}$.

Ask them to justify their reasons for ordering the cards as they did.

Number Lines

Organize students into groups. Provide students with equal lengths of cash register tape and ask them to fold or mark the strips into fractional parts. Have groups tape their fraction strips together to make separate number lines for halves, thirds, quarters, and so on. Then, ask them to add labels, for example: $\frac{0}{5}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{5}{5}$ (or 1), then $1\frac{1}{5}$, and so on. Have students use their number lines to count in fractions. For example, say: Begin at one-third, then count on by two-thirds. Encourage students to compare strips to make other counts.

Say: Begin at one and a quarter and count in halves.

Grades 5-8: ★ ★ ★ Major Focus

Places on a Number Line

String up a clothesline across the classroom. Add a card labelled "0" at one end and a card labelled "1" at the other end. Ask students to determine where fraction cards would be positioned on the line and justify their suggestions. Draw out the idea that there is a much greater difference between and, for example, than there is between $\frac{32}{33}$ and $\frac{33}{34}$ in order to help them understand that must be closer to 1 than

Spending Money

Pose this problem to students: Felicity and Cameron both got money as birthday gifts. Felicity said she spent $\frac{1}{4}$ of her money. Cameron said he spent $\frac{1}{5}$ of his. "You spent more than me!" Felicity added. Cameron replied, "I couldn't have, a fifth is less than a quarter." Ask: Could Cameron be right? How could that happen?

Counting Fractions

Pose this problem to help students count in fractional amounts: I need $1\frac{1}{2}$ m of ribbon to make a bow for a present, but I only have a $\frac{1}{10}$ m ruler. How would I count to measure the ribbon I need? Have students record the count on a number line.

Comparing Fractions

Ask students to compare two fractions, such as $\frac{2}{3}$ and $\frac{4}{5}$. Ask: Which number is larger? How do you know? Use a number line to prove that your answer is correct.

Fraction Problems

Pose this problem to students: Each day, a baker uses $\frac{3}{8}$ of a bag of flour to make bread, and $\frac{1}{4}$ of the same bag of flour to make cakes. Is more flour used to make bread or cakes? Have students use diagrams to show a partner which quantity is bigger.

Sorting Fractions

Have students explore the relative size of fractions by sorting fraction cards into given categories. For example: less than one or more than one; nearer to zero or nearer to one; nearer to zero, nearer to half or nearer to one. Encourage students to use materials or diagrams to justify how they have sorted their fraction cards.

Fractions on a Number Line

Ask students to use a number line, marked from 0 to 50, to indicate the position of fractions as numbers as well as fractions of numbers. For example, say: Show the number $\frac{7}{8}$. Show the number that is $\frac{7}{8}$ of 16. Have students compare the ,language used when referring to fractions as numbers and fractions as operators. Discuss the identity of the whole in each context.

Estimating

Have students use equal lengths of cash register tape to estimate (without folding to check) the position of a different fraction on each strip. For example, one-third on the first strip, one-sixth on the second, five-sixths, three-ninths, seven-eighths, and so on. Then, ask students to place their strips of paper onenext to the other and review their decisions, making changes to their estimates if necessary. Encourage them to check and try other fractions to improve their estimates.

Sample Learning Activities

Grades 3-5: ★ ★ Important Focus

Making Jello

Students make jello (lemonade) using different ratios of water to jello crystals (lemonade concentrate) and say which ratio makes the best jello (lemonade). Say: I have lost the instructions for how to make jello (lemonade). How can we find out the right proportions to make jello (lemonade)? Have students test different concentrations and then decide how they can use fractions to represent the different concentrations. For example, students might say: *I thought the best jello was made with one package of jello crystals and 500 mL of water, so that is* $\frac{1}{500}$. Invite students to say what each of the numbers in the fraction represents.

Feeding the Class

Extend *Making Jello*, above. Ask students to work out how much water and jello crystals are needed to make enough jello for the whole class or school. Ask: How can we use the fraction to help us work this out?

Proportional Quantities

Encourage students to use their own methods to solve simple problems involving proportional quantities. For example, say: When visiting the zoo, a kindergarten class needs one adult for every five students. One adult per ten students is needed for the rest of the school. How many adults are needed for the whole school to visit the zoo? Invite students to describe and compare their approaches and then help them to see how the fractions $\frac{1}{5}$ and $\frac{1}{10}$ can be used.

Proportional Relationships

Ask students to use fractions to represent proportional relationships. Students' answers might include: *I got seven out of ten right in my spelling test, so that makes it* $\frac{7}{10}$. *Half of the class like to eat chips, so that is* $\frac{15}{30}$ *or* $\frac{1}{2}$. Encourage students to read their fractions as, for example, seven out of ten. Ask: Why doesn't it make sense to read this as "seven-tenths"?

Bargain Hunting

Discuss and explore with students situations such as: If you bought a pair of jeans at a "25% off" sale, what fraction of the full price would you save? What if it was 50% off? What fraction of the full price would you save?

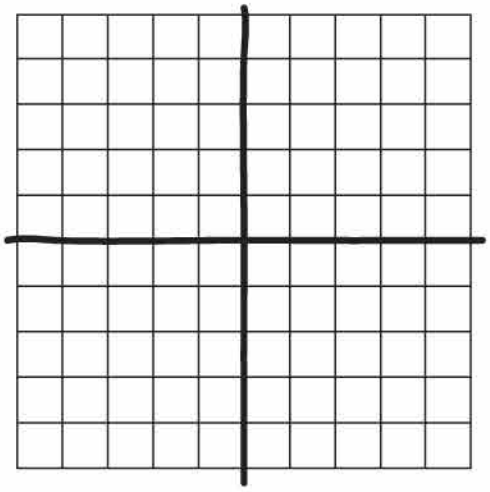
Grades 3-5: ★ ★ Important Focus

Matching Games

Have students make sets of playing cards composed of pairs or sets of matching common fractions and decimals. Include equivalent percentages in the cards; for example, half, $\frac{1}{2}$, 0.5 and 50%. Then, have students use the cards to play matching games. (See *Concentration*, page 156, and *Matching Games*, page 159.)

Hundred Square

Ask students to use 10 x 10 arrays on grid paper to make equivalent fractions in order to find percentages. For example, to find $\frac{1}{4}$ as a percentage, students use 100 squares, share them out into four groups and then say how many squares in each group. Ask students to represent this as a fraction out of 100 ($\frac{25}{100}$) and read it as 25 "out of 100". Draw out the idea that the "%" sign is used to show a ratio "out of 100". Then, have students convert the fraction into percentage notation (*25%*). To find $\frac{3}{4}$ as a percentage, students might say how many squares in three of the groups. Have them represent this as a fraction out of 100 and then rewrite it as a percentage. To find $\frac{12}{20}$ as a percentage, students might share the 100 squares out into 20 groups and say how many are in 12 of them. Have students represent this as $\frac{60}{100}$ and read it as "60 out of 100". Then, have students write this as "60%".



Sample Learning Activities

Grades 5-8: ★ ★ ★ Major Focus

Discounts

Have students investigate questions, such as: Would you rather have $\frac{1}{3}$ off the price of something or a discount of 30%? What's a better deal: $\frac{1}{5}$ off the price of something or 50% off? Ask students to justify their choices.

Units of Measurement

Ask students to investigate the ratios used to represent proportional relationships between different units of measurement. For example, a cookie recipe might require two scoopfuls of flour and six spoonfuls of butter. Students could express the relationship as $\frac{\text{flour}}{\text{butter}} = \frac{2}{6}$. Then, they could generate equivalent fractions to find, for example, how many spoonfuls of butter they would need for four (six, one) scoopfuls of flour. Ask: How many spoonfuls of butter would be needed for seven scoopfuls of flour?

Proportional Relationships

Give students a selection of newspaper and/or magazine articles. Then, ask them to investigate the accuracy with which proportional relationships are expressed in percentages, fractions or decimals. For example, say: This newspaper article states that there was a 34% drop in enrolments at universities in 1998. What might this mean? How would the reporter have calculated this figure? What would we need to know to check this? Does the article give us enough information to do this? If there is not enough information in the article, have students write to the editor and ask for all the data. Invite students to display and share their analyses.

Sensible Fractions

Pose some problems and have students say when adding fractions makes sense. For example, say:

- Brett ate $\frac{3}{8}$ of the pizza yesterday and $\frac{4}{8}$ today. What fraction of the pizza has he eaten?
- Brett got $\frac{3}{8}$ of the spelling words correct yesterday and $\frac{4}{8}$ today. What fraction of the words has he spelled correctly so far this week?
- $\frac{3}{8}$ of the girls and $\frac{4}{8}$ of the boys walk to school. What fraction of the students walk to school?
- My orange drink was $\frac{3}{8}$ juice to water, and my sister's was $\frac{4}{8}$. What would the concentration of juice be if the drinks were combined? Is it $\frac{7}{8}$, $\frac{7}{16}$ or neither? How do you know? Why can't you add some fractions?

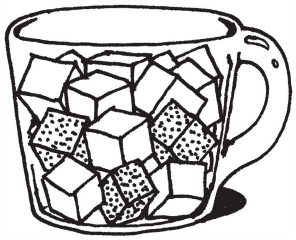
Grades 5-8: ★ ★ ★ Major Focus

Ratios

Invite students to use cash register tape models to represent ratio situations. For example, say: It takes Andrew five steps to cover the same distance as his dad covers in three steps. Have students work out the ratio of Andrew's steps to his dad's and represent this with a fraction ($\frac{5}{3}$). Ask: How many steps will Andrew have taken when his dad has taken six (nine) steps? Is the ratio of Andrew's steps to his dad's steps still the same after six steps? Later, have students say why the ratio was written as $\frac{5}{3}$ and not $\frac{3}{5}$. Ask: What would the ratio represent if it was written as $\frac{3}{5}$?

Making Juice

Explore dilution ratios with students using white and orange linking cubes. The white cubes represent water; the orange cubes represent orange juice concentrate. Invite students to mix different strengths of "juice". Begin with a mixture of three orange and nine white cubes. Ask: If I want to keep the taste the same and make more (less) of this drink, what could I do? Have students investigate the pattern, leading to the underlying ratio of one-third: one-part concentrate to three-parts water. Do the same with a $\frac{15}{20}$ proportion of concentrate to water. Ask: How could I make more (less)?



Combining Proportions

Have students decide whether it is sensible to combine proportions. For example, ask: If half the students in one class and a third of the students in a second class are girls, what fraction of the two classes combined are girls? Does it make sense to add a half and a third in the usual way? Why? Why not? Would it make more sense to say one in two students in one class and one in three students in the second class are girls, so altogether two in five ($\frac{2}{5}$) students must be girls? Compare this with the fraction obtained by adding all the girls together for the numerator and adding all the students together to find the denominator. Ask: Are the fractions the same (that is, equivalent)? Why not?

BACKGROUND NOTES

Learning Basic Facts

Students should not be expected to try to memorize facts they do not understand. Equally, however, understanding where the basic facts come from and having worked them out for themselves is NOT enough to enable students to remember them. Students usually do need some drill with number facts if they are to be able to readily recall them. What is needed is a rational rather than a rote approach to learning the basic facts.

Addition Facts

Students might first discover and record the addition combinations to ten, convincing themselves that particular number facts "always work". They can often do some of these by counting in their mind's eye (three and two more) and/or quickly checking using their fingers, as well as by using materials. They should develop organized lists showing the numbers that fit together (part-part-whole) to make five or eight or ten. Once there is a meaningful basis for these "facts", students need focused practice on remembering them—small amounts at frequent intervals.

Meanwhile, many of the combinations to 20 should be being established using materials and diagrams. Students should be using mental arithmetic to extend the facts they already know and remember. For example: *I don't know 7 + 5 but it is*  *, which is two more than 5 + 5, so it must be 12; 8 + 6 is like two sevens so it must be 14.*

Addition Facts

+	1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10	11
2	3	4	5	6	7	8	9	10	11	12
3	4	5	6	7	8	9	10	11	12	13
4	5	6	7	8	9	10	11	12	13	14
5	6	7	8	9	10	11	12	13	14	15
6	7	8	9	10	11	12	13	14	15	16
7	8	9	10	11	12	13	14	15	16	17
8	9	10	11	12	13	14	15	16	17	18
9	10	11	12	13	14	15	16	17	18	19
10	11	12	13	14	15	16	17	18	19	20

Once the combinations to 20 have been discovered and recorded, students should be introduced to the use of a two-way table to record the facts (See Appendix: Line Master 18).

Students will need to learn how to read the table and should investigate patterns in it. They should note the sums on either side of the diagonal are in a sense "the same" and that the number of facts to be remembered is almost halved when we use the commutative property of addition (55 instead of 100). The "double numbers" on the diagonal (2, 4, 6, 8, 10, ...) are helpful in a lot of contexts and students' attention should be drawn particularly to this sequence of numbers.

Multiplication Facts

The usual approach to learning multiplication facts is to learn to chant through the multiplication facts in order. Students learn the two times "table", the three times "table", then the four times "table", then the fives, and so on. While students do need to memorize the basic multiplication facts, learning them by chanting tables is not a particularly helpful approach, for the following reasons.

- Firstly, many students who have learned their "tables" in this way have difficulty remembering the facts without chanting through the table. Hence, it hinders the development of instant recall rather than helping it.
- Secondly, setting out the multiplication facts in columns and learning each set of tables separately masks the commutative property. Therefore, many students who *do* know, say, six fives, do not relate it to five sixes and have to remember almost twice as many facts as they need to.
- Thirdly, other patterns, such as that six times is double three times, are masked, which also increases the memory load for students.

It is likely to be much easier for students to remember basic facts if they practise them in clusters that help work them out. A possible sequence could be as follows:

Build up the facts to 5 x 5

Start with the twos (doubles), fours (double doubles), fives (because of the easy patterns and the links to our fingers) and then the threes. Put these in a two-way table. Commutativity reduces the 25 facts to be remembered to only 15, and if we remove the ones, there are only

ten to remember. Use the 5 x 5 table to show students that by learning just ten facts they "get" 25! Consolidate these facts and build up speed of recall with frequent short periods of practice.

X	1	2	3	4	5
1	1	2	3	4	5
2		4	6	8	10
3			9	12	15
4				16	20
5					25

Note that students may, at the same time, be able to skip count in twos, threes, fours and fives well beyond these facts. However, this requires them to work their way through the skips. This is not the same as being able to immediately recall 4 x 3, which is what focused drill should help them to do.

The ones and twos

At this stage, students could focus on the notion of "doubling" and build their capacity to readily find "double a number". (This should be linked to the diagonal of the addition table and to the notion of even and odd numbers in order to help students make connections between related mathematical ideas.)

The tens

Focus on groups of ten and counting in tens. Help students become convinced about why three tens are the same as ten threes and practise these together. There will be ten facts to remember, but the pattern makes them easy and students will readily recall them.

The squares

Many teachers also find that explorations of patterns of squares help students learn the squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100. It seems students "like" the square numbers and learn them fairly readily. Thus, facts such as 7 x 7 may be learned earlier than other related facts. Students should develop instant recall of these facts.

Students have now developed the facts from 1 x 1 to 5 x 5 and the facts involving one, two and ten and the squares. Help them make a two-way table (See Appendix: Line Master 19) in which they record the multiplication facts they now know.

Multiplication Facts

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15					30
4	4	8	12	16	20					40
5	5	10	15	20	25					50
6	6	12				36				60
7	7	14					49			70
8	8	16						64		80
9	9	18							81	90
10	10	20	30	40	50	60	70	80	90	100

There are various orders in which the remaining "facts" can be developed. Some teachers find it most helpful to move out in an ever-increasing square, so that the next cluster of facts is those to fill in the square to 6 x 6. This requires the addition of three new facts (6 x 3, 6 x 4, 6 x 5) and their "partner" facts (3 x 6, 4 x 6, 5 x 6). Then move out to 7 x 7 and so on.

An alternative sequence could be as follows.

The five facts

Build up the five facts to 5 x 10, noting the relationship to the ten facts (five eights is half of ten eights; half of eight tens is four tens or 40; so 5 x 8 is 40) and the pattern in the units digits. Add to the table and memorize.

The four facts and eight facts to 8 x 8

First build on the "doubles" or twos (2 x 1 to 2 x 10) to get the "double doubles" or fours (to 4 x 10). Add these to the table. Double the four facts to produce the eights up to 8 x 5 (if you know 4 x 3, you can double to get 8 x 3). Add these to the table. Use commutativity to work out the additional facts (if you know 8 x 3 you also know 3 x 8). When first practising these facts, give students plenty of time to work them out mentally using the doubling strategy or some other method they prefer. Gradually build up speed to get "instant recall". Use various doubling and other patterns to build up the extra eight facts (8 x 6 to 8 x 8). Practise to memorize.

The three facts and six facts

Build up the three additional three facts (3 x 6, 3 x 7, 3 x 9) and add to the table. Double these to get the sixes or use other known facts (six fours is five fours and four more). Five new facts to learn and their partners give you ten more.

The nine facts

Build on the threes and the sixes using number partitions to add the additional two nine facts and their partners. Although most are known, revisit the nines to link them to 10 – 1, so that students see that 9 x 7 is ten sevens take away seven. Initially, allow students time to do the calculation using mental arithmetic strategies and gradually help them memorize for speedy recall.

The seven facts

The seven facts are all known!

This is not the only possible sequence. The important thing is to assist students to use rational thought processes rather than rote memory to learn the facts.

Reducing the Stress

If students have reached grades 5 to 6 and are struggling to remember the multiplication facts, they may have built up some anxiety about them. Often it is worth spending time explaining how to read a 10 x 10 multiplication table and then having them systematically work through the table, crossing out those they know.

Everyone can draw a line through the "one times" and the "times one". Most know the doubles and the fives and tens. Quite a lot will know the squares. Spend some time on the commutative property. Students do not need to remember the word, but they should be able to say and understand why: *If I know 8 x 5 then I also know 5 x 8.* Show students how this reduces the number of facts to be remembered.

Remarkably, after removing the ones, twos, fives and tens and the squares, only 15 of the 100 facts remain. Emphasize that they are almost there! Most students will be able to cross off at least some of these 15.

Students can then make personal "prompt" cards for their remaining unknown facts. Have them set a personal target of, say, three to learn this week. During the week, help students work out how their three target facts relate to other facts they know. Periodically through the week, their partner should test them on the facts. When they have correctly recalled a fact, say, ten times, they cross it off their multiplication table list and store the card for later re-testing. Over

the next several weeks, their partner should test them on previous weeks' facts, as well as their targeted three for this week.

Techniques for Mental Calculation

Students need practice with a wide range of strategies for calculating mentally. Mental arithmetic is flexible, purposeful and personal so it cannot be made routine. For example: to add 99 to 125, a sensible strategy would be to add 100 and subtract one; to add 64 to 125, you might add 60 and then four; to add 64 to 96, you might add the four and then the 60.

The fact that the calculation is done mentally does not mean that the presentation is always oral. Often we add a string of numbers that we can see as when playing Scrabble. When we add the digits in each column for column addition, the sums often go well beyond the basic facts and so mental arithmetic is needed. In some cases, students (like adults) will use some informal jottings on paper to help keep track of their thinking. Recording partial answers is widely used by adults and should not be discouraged. The choice is not between fully mental approaches and standard written approaches. The goal is flexibility and efficiency rather than standardization.

Students should use place value to extend the range of calculations they carry out mentally. For example:

- Count backwards and forwards in tens: 10, 20, 30, 40, ...
- Count in tens from any starting point: 14, 24, 34, 44, ... and 53, 43, 33, ...
- Add in tens, twenties and thirties, hundreds and so on, from any starting point: 23, 43, 63, ...
- Generalize basic facts: 8 + 7 = 15 so 18 + 7 = 25, 28 + 7 = 35; 6 x 7 = 42 so 60 x 7 = 420

The properties of the operations (when multiplying several numbers, the order does not matter), the relationships between them (division is the inverse of multiplication), number partitions and place value form the basis of the following mental calculation strategies.

Use relationships (commutativity and inverses)

- Adding: order does not matter. **4 + 27** is 27 + 4, so 28, 29, 30, 31.
- Multiplying: order does not matter. **24** twos is **2** twenty-fours, so 48.
- Subtracting: thinking of an addition might help. **13 – 8**, think "eight add what is 13?"
- Dividing: thinking of a multiplication might help. **63 ÷ 9**, think "how many nines make 63?"

Compensate (partition and rearrange)

- Add: take some from one number to give to the other. **8 + 7** is 10 + 5; **68 + 37** is 70 + 35
- Multiply: take out a factor from one to give to the other. **15 x 6** is 15 times 2 times 3, so 30 times 3, so 90.
- Subtract: change the numbers by adding or subtracting the same amount. **62 – 37** is 65 – 40.
- Divide: change the numbers by multiplying or dividing by the same amount. **29 ÷ 5** is 58 ÷ 10.

Use compatible numbers and bridge

- Making change: **100 – 68**. Think "100 and what fits with 68" OR "It cost 68 cents. What's the change from $1?"
- Rearrange the order: **8 + 7 + 2** is 8 and 2 is 10 plus another 7 is 17; **68 + 27 + 12** is 68 and 12 is 80 plus 20 is 100 plus seven, so 107.
- Bridging: **9 + 4** is 9 + 1 + 3 = 10 + 3 = 13; **68 + 47** is 68 and 32 will make 100 and 15 left, so 115.

Front load (start with the biggest place)

- Bring on the tens and then the ones: **28 + 37** is 38, 48, 58 and 7 more, so 60, 65.
- Do both tens and then both ones: **68 + 37** is 90 + 15, so 100 and 5 more, so 105.

Imagine a number line

- Jump along or back: **364 – 198**: starting at 198, it takes 2 to get to 200 and another 164 to get to 364, so 166; OR starting at 364 go back 64 to 300, 100 more to 200 (so that's 164) and back 2 more to 198, so 166.

Multiply in parts (partition and multiply the parts)

- Round a number and adjust: **7 x 9** is 7 tens take away 7 ones, so 70 – 7, so 63; **99 x 6** is 600 take away six.
- Use place value partitions: **6 x 25** is 6 x 20 add 6 x 5, so 120 add 30.

Use factors

- Double, double, double: **4 x 14** is double double 14, so double 28, so 56.
- Change to a multiplication you know: **3 x 18** is 3 times 3 times 6, so 9 times 6, so 54.

- Multiply by five: **5 x 8** is 8 fives, which is 4 groups of 2 fives, 4 tens so 40.
- Multiply by fifty: **50 x 72**; 50 is half of a hundred, so half of 72 hundred, so 36 hundred or 3600.
- Multiply by twenty-five: **36 x 25**; Notice the 25 and look for 4 to make 100, so 9 x 4 x 25 or 900.
- Doubling and halving: **45 x 14** is the same as 90 x 7, so 630.