Grade 2/3

Number Sense

Assessment Package

SNAP Assessment

First Steps in Math

Diagnostic Assessments & Learning Activities

SD71-Comox Valley Schools, 2004

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SD71 K-3 Numeracy Assessment

Early Years Foundations

Early Years teachers identify and monitor their learners' strengths and needs to ensure continuous growth in numeracy. This assessment tool is designed for classroom teachers and focuses on critical skills that foster numeracy development. The tool can be used to inform instructional decisions as part of the ongoing instructional cycles throughout the Early Years K-3 and is useful for school data collection to determine school goals and to pass on to future teachers in support of transitions.

Wiliam and Leahy (2015, p.9) state that many in education talk of 'data driven instruction' resulting in large scale assessments that provide information on our learners after the fact – too late to do anything about it! We should instead be focusing on 'decision-driven data collection' answering the questions:

- "What do you want to know about your learners?"
- "When do you want to know it?"

We need 'laser-beam focused' assessments to help us reach all our learners that are quick to implement, interpret and act upon.

NOTE: Please note that these assessment tools focus on some of the key numeracy areas and do not represent a comprehensive numeracy learning program. For more information about our district's holistic approach to numeracy learning, please see the SD71 Numeracy Framework.

K-3 Numeracy Developmental Stages

Counting is the landscape of learning for Early Primary. Students work to gain mastery over the **Principles of Counting** and develop their counting skills along this trajectory:

Principles of Counting and Developmental Stages

	Stage	Counting Skills
1	Precounter	No counting, says 'ball'
2	Reciter	Says numbers, not in the correct order
3	Corresponder	1:1 matcher, needs to recount if asked, 'How many'?
4	Counter	Counts if set is organized; has cardinality
5	Producer	Can count objects to a target number: "Get me 5 blocks."
6	Counter and Producer	Can count random groupings, keeping track of the count
7	Counter Backwards	Removing objects, 'countdown'
8	Counter from Any Number	Counting on
9	Skip Counter	Understands we get the same answer counting by groups as by ones

(Adapted from *Van de Walle, 2022*)

- ✓ Once children can meaningfully count, they need to build **number relationships** to develop understanding for the operations (+/-). This is the time we build the benchmarks of 5 and 10 and develop flexible strategies for composing and decomposing numbers.

- ✓ Place value begins to develop within the early years. Students in K/1 are **unitary** thinkers. They only count by ones, and they see ten as a collection of ones – they are not yet able to think of ten as a single unit.

- ✓ By grades 2/3, students begin counting a group of ten objects as a single item – **unitizing** – and this fosters the development in trusting the count in skip-counting. Students begin counting by tens **and** ones and are moving toward using these groupings in computations.

What's on the Horizon?

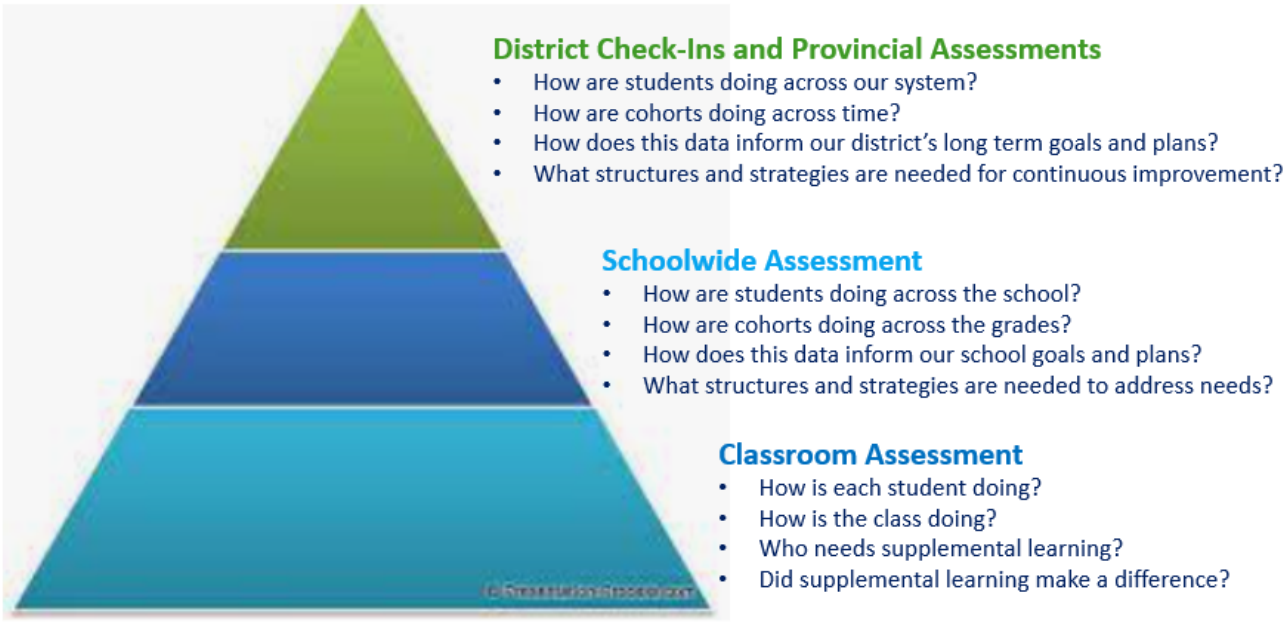
✓ As students move into the Intermediate level, they are leaving the realm of **additive** thinking and entering the landscape of **multiplicative** thinking, using the power of 'groups' in both multiplication & division.

✓ They are also seeing the patterns in place value as they extend both ways – from thousandths up to millions.

✓ The new landscape of learning is now rational numbers - fractions and decimals – and the new thinking that emerges when learners realize there is *infinity* in the space between whole numbers, that they once thought contained…nothing. A big idea with which to grapple!

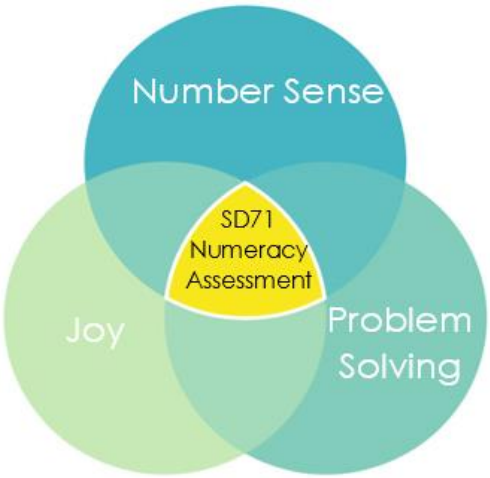
Assessment Design for the Frameworks

Grounded in Classroom Assessment System



District Numeracy Check-In Points

With in the SD71 Numeracy Framework, there are district check-in points to monitor how learners in our system are doing across the years and to inform decision making regarding numeracy initiatives, professional learning opportunities, and resources. The district will be extracting the data from both the SNAP and Problem-Solving Assessments that classroom teachers have entered in Grades 3, 5, and 8.



District Check-In Points and Provincial Assessments Timeline:

Number Sense Numeracy Assessments:

1. **Whole class**: SNAP (Student Numeracy Assessment and Practice) three to four times a year.

2. **Targeted Diagnostic Assessment**: First Steps in Math - Targets misconceptions and gaps, accompanied with learning activities to build key math understandings for the whole class, small groups, and/or one-to-one instruction.

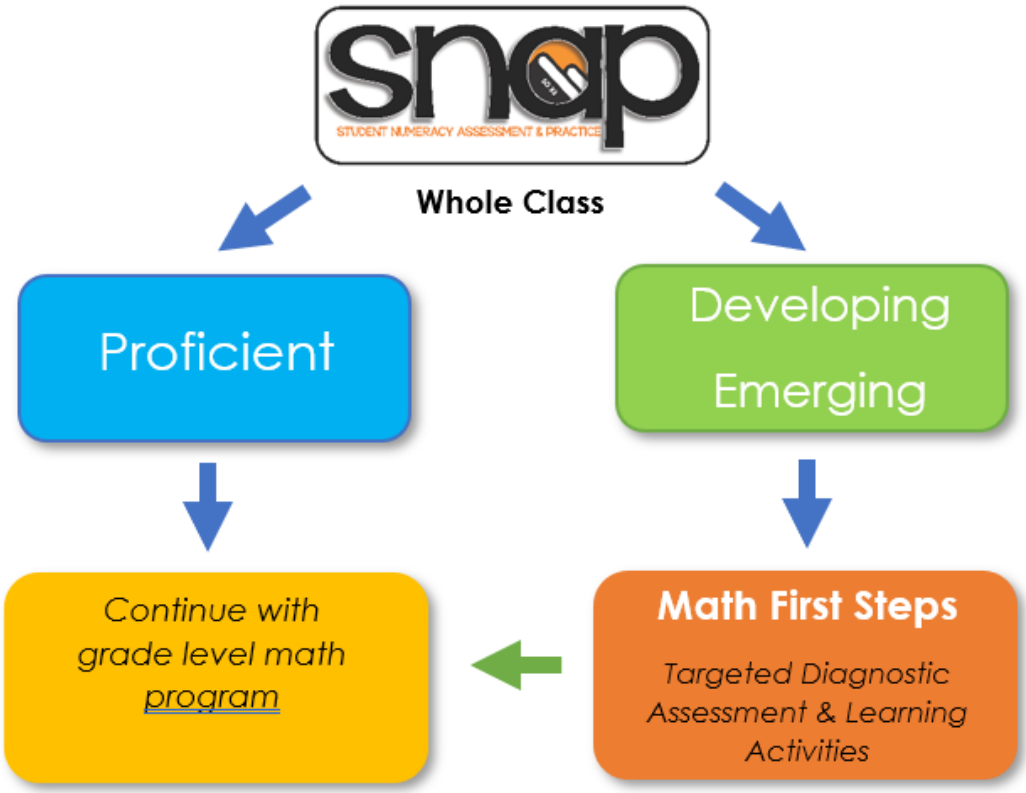
Mathematics is not about numbers, equations, computations, or algorithms: it is about understanding.

William Paul Thurston

3. **Daily Observations**: Can be recorded in checklists, comments, and reflections.

We value what we measure.

Do we measure what we value?



Whole Class Assessment SNAP K-3:

The Student Numeracy Assessment and Practice (SNAP) is the Chilliwack district numeracy assessment for all students in grades K – 7. It was created by a group of Chilliwack educators and has been used in all grades K – 7 classes since September 2016. The SNAP is a unique assessment; not only is it a measurement of achievement, but it is intended to be used as a practice tool throughout the entire year. The data it provides can be used to inform and guide instructional planning.

The SNAP is a two-page assessment that focuses on the foundational skills of mathematics: Number Sense and Operations. It **compliments any balanced math program** and quickly provides teachers the information they need for responsive planning and instruction.

SNAP is fully aligned with the BC Curricular Competencies in math. Each area of the assessment is connected to a particular competency, and the competencies are built right into the grading rubric.

(SNAP Teacher Guide – updated, Chilliwack School District).

NOTE: Please reference the SNAP website for teacher guides, templates, rubrics, and exemplars for your grade.

SNAP - Identifying who may require targeted instruction.

Use the SNAP with the **whole class** and sort completed assessments into 2-3 groups – students who have demonstrated ***mastery/proficiency*** and those who have not. You may want to break the second group down into 'developing' and 'emerging' groupings if appropriate.

SNAP Kindergarten: Number Sense to 10
SNAP Grade 1: Number Sense to 20
SNAP Grade 2: • Number Sense to 100 • Operations: Addition
SNAP Grade 3: • Number Sense to 1000 • Operations: Subtraction

Targeted Diagnostic Assessment: First Steps in Math – Number Sense

It is this second group (emerging/developing) with whom we use the set of rich ***First Steps in Math - Number Sense*** diagnostics to identify learners' misconceptions and gaps and to determine which learning activities can address these and build their understanding. The *First Steps in Math* resource is designed to pinpoint and target student misconceptions and can be used to inform next steps in learning. This package includes the curated diagnostic tasks from the resource.

It is recommended that the teacher focus on **one** diagnostic assessment at a time, followed by an analysis to determine the next step for supplemental learning. "Next step" learning activities that correspond with each diagnostic are also provided to target the misconception or key understanding. All activities on the *Learning Activities* sheet will support the same misconception starting with less complex activities moving toward more complex activities. If the suggested learning activity does not seem to work for learners, try another corresponding activity. Professional judgement and relationship with your students will guide you in determining what works better for them.

Note that both the diagnostic assessments and learning activities can benefit the whole class or can be done during small group and/or one-to-one instruction.

Frequently Asked Questions:

1. **Can I have my LST or CST assess my students for me?**

The SNAP and First Steps in Math numeracy assessments are designed for classroom teachers to use as part of their teaching, learning and assessment cycle. It is important for teachers to know where their students' strengths and stretches are in key *num*eracy skills to inform planning and to scaffold learning to meet the needs of their learners.

2. **Do I have to assess students all at once?**

Teachers assess at a variety of times in a variety of ways*, depending on assessment purposes and what works for their students*. *As described herein, t*he SNAP can be performed as a whole class, while First Steps in Math *resources can help to identify specific misconceptions and gaps as well as provide suggestions for targeted instruction to address them.*

3. **Do I have to use this assessment if I have my own numeracy assessment tools?**

Teacher colleagues in our district have reviewed and discussed many possible strategies and assessments for number sense learning and have selected SNAP and First Steps in Math based on research and what works with students. It is recommended that *intermediate and middle years* teachers become familiar with the common language and understandings of these district wide assessment tools, as they will be used throughout K-9, and support student numeracy learning across the school years.

4. **What do I do if my students are not demonstrating proficiency on a skill?**

Teaching, Learning and Assessment Cycle:

i. Whole class teaching, and repeated experience practicing the skill
ii. Assess
iii. Targeted classroom instruction according *to identified* student needs.
iv. Assess
v. Targeted classroom instruction and/or consult with school-based team about other interventions that may be necessary.
vi. Assess

Frequently Asked Questions:

6. **How will the information collected from this assessment be used?**

Classroom Teachers use this information to inform instruction and monitor progress of their students' numeracy skills. Additionally, this information will be helpful to pass on to the next *year's* teacher.

The School can use the data for school growth plans, allocation of resources and *to* inform school-based team discussions and strategies.

The District will collect the SNAP- Number Sense *and* Operations information *in Grades 3, 5, and 8*. This data will be used to inform decisions regarding resource allocation and professional learning opportunities, as well as to monitor the numeracy development of student cohorts over time.

Bibliography

First Steps in Mathematics.(2007-13). Pearson Canada.

SD38 - Chilliwack School District (retrieved 2024). *SNAP: Student Numeracy Assessment and Practice*.

Wiliam, D. and Leahy, S. (2015). *Embedding formative assessment: Practical techniques for K-12 classrooms* Learning Sciences International.

Grade 2/3
SNAP Assessment

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TEACHER GUIDE

What is the SNAP?

The Student Numeracy Assessment and Practice (SNAP) is the Chilliwack district numeracy assessment for all students in grades 2 – 7. It was created by a group of Chilliwack educators and has been used in all grades 2 – 7 classes since September 2016.

The SNAP is a unique assessment; not only is it a measurement of achievement, but it is **intended to be used as a practice tool throughout the entire year.** The data it provides should be used to **inform and guide instructional planning**. If only used as a summative assessment, the SNAP will not help in achieving one of our main goals, which is to improve students' proficiency in number sense and operations.

The SNAP is a two-page assessment that focuses on the foundational skills of mathematics: Number Sense and Operations. It compliments any balanced math program and quickly provides teachers the information they need for responsive planning and instruction. **Access the SNAP Number Sense and Operations templates under the SNAP Templates tab on the website.**

SNAP is fully aligned with the BC Curricular Competencies in math. Each area of the assessment is connected to a particular competency, and the competencies are built right into the grading rubric. **Access the grading rubrics under the SNAP Training tab on the website.** The rubrics are the same for all grades. It is a good idea to participate in collaborative marking with colleagues to help establish common expectations.

How to Effectively use the SNAP

SNAP practice does not always need to be on the SNAP templates; in fact, once areas of need are identified, most number sense and operations practice will happen through other strategies, such as daily high yield number sense routines (e.g. number talks, count around the circle) and whole or small-group instruction. **Find resources that support each of the four curricular competencies under the Resources tab on the website.** Explore the Recommended Links for sites that support the teaching and learning of number sense and operations.

Curricular Content and Competency Areas

While the SNAP templates and rubrics are the same for grades 2-7, the curricular content and competency goals (pulled directly from the BC Math Curriculum) change and follow a spiraled approach. The table below outlines the curricular areas that students will be assessed on at the end of May. The goal is that all students be proficient (3 on the rubric) in their grade-level standards by the end of the school year. The examples given in the Operations sections are examples of year-end appropriate operations. **There are no district-prescribed numbers or operations for the year-end assessment, but at the request of teachers, numbers and operations have been suggested below to provide guidance.**

Grade	Operations - Sample operations	Number Sense - Sample numbers	Number Sense – Skip counting sample numbers
2	Addition of two-digit numbers without regrouping 24+33 51+17	Number concepts to 100. Any two-digit number. 42 67	Count forward by: 2, 5, or 10 Count backward by: 2
3	Subtraction of three-digit numbers with regrouping 427-153 754-226	Number concepts to 1000. Any three-digit number. 327 568	Count forward by: 4 or 20 Count backward by: 3 or 5
4	Multiplication of a one-digit number by a three-digit number. 4x326 7x142	Number concepts to 10000. Any four-digit number. 5904 6138	Count forward by: 6s or 300s Count backward by: 4 or 25
5	Division of a three-digit number by a one-digit number with a remainder. 635÷3 291÷4	Number concepts to 1000000. Any six-digit number. 347075 762346	Count forward by: 7 or 250 Count backward by: 6 or 30
6	Division of four-digit decimal number to hundredths. Quotient should not exceed thousandths. 47.35÷5 71.76÷3	Number concepts thousandths to billions. Any decimal number to the thousandths. 45.892 534.21	Count forward by: .12 or 5000 Count backward by: .6 or 14
7	Percentage calculations. Find the percent of a number. Answer should be in the tenths or hundredths. 16% of 85 47% of 42	Integer concepts. Any negative two-digit whole number. -23 -75	Choose numbers that will make students count through 0. Count forward by: 4 or 12 Count backward by: 5 or 20

Remember that the SNAP templates are intended to be used throughout the year for any numbers or operations in your curriculum.

When introducing your students to the SNAP, take your time and explicitly teach and model each component of the assessment. Use content that the students should be confident with from previous years. You can chunk the assessment into smaller pieces. **The Zoom into SNAP templates under the Resources tab on the website chunk the assessment by competency**. You can complete SNAPs as a whole group guided activity and have students work with partners to help build confidence. Have students share their thinking; encourage them to use many different ways to demonstrate their thinking and solutions.

Remember that the SNAP templates are intended to be used throughout the year for any numbers or operations in your curriculum.

The SNAP templates

Access templates under SNAP Templates tab.

NUMBER SENSE:

See Grading Rubrics for specific criteria.

DRAW: The picture must show the value of the number. A written explanation or a legend should be included in the "write to describe your picture" box.

SKIP-COUNTING: Begin at the number and count forwards and backward by numbers chosen by the teacher. *Update – Spring 2024* Teachers have requested guidance on appropriate numbers to use in this section for the May assessment. We have provided sample numbers based on the curriculum at each grade in the table above.

EQUATIONS: Students who are demonstrating full proficiency will be using grade-appropriate operations in their equations. Teachers should be very specific about their expectations in this section to avoid students using equations like 4561+1=4562, for example (which is not a grade-appropriate operation in Gr. 4).

REAL-LIFE EXAMPLE: The examples must be realistic and specific. It is important that students demonstrate an understanding of value in their example. For instance, "Wayne Gretzky's number is 99" does not show an understanding of value; "we have 99 grade three students in our school" does. Literature and sharing out of real-life examples helps students to make connections to the numbers and add to their bank of knowledge. There is an excellent list of math picture books on the Coast Metro Elementary Math Project site.

NUMBER LINE: For grades 2-5, the endpoints to the number line are provided. For grades 6 & 7, the students choose their own endpoints according to the number chosen for the assessment. To demonstrate full proficiency, students will add at least three benchmarks to their number line to help situate the number. Clothesline Math is an excellent routine to help students to become more proficient with number lines.

REFLECTION: Reflections help increase the value of a learning experience. They allow students to link ideas and construct meaning from their experiences. Students should have opportunities to reflect on their learning at the end of every lesson. Explicit teaching about how to reflect effectively will improve the quality of student responses in this section; reflection sentence stems are available in the Connecting and Reflecting Resources page.

OPERATIONS

See Grading Rubrics for specific criteria.

ESTIMATE: Students will learn to value the skill of estimating through discussions about real-life situations where a person would typically estimate rather than calculate. In which situations would one prefer a high estimate? A low estimate? Explicit instruction on estimation strategies will allow students to select and use an appropriate strategy for the given operation.

DRAW: Students will visually represent the operation. The visual may or may not contain the solution to the operation. Consider the use of bar diagrams as an appropriate, proportional model for the operations. Simply replacing the numbers in the operation with a base ten representation does not demonstrate an understanding of the operation.

CALCULATE: Multiple grade-appropriate calculations demonstrate proficient achievement. Students are not required to use the standard algorithm for any operation. Using the reverse operation to "check" their work is also a recommended strategy. Refer to your grade-specific curriculum elaborations for suggested alternate computation strategies.

REAL-LIFE EXAMPLE OR WORD PROBLEM: Students will provide details on a real-life situation where the given operation would be used to find an amount. Look for evidence that communicates their understanding of the use of the operation. For example, if the operation was 316-141 a student could suggest, "there were 316 blueberries on the bush and I picked 141 of them." For the teacher to know if they understand what the difference between 316 and 141 represents in this situation, they should add, "How many blueberries were left on the bush?"

Grade 2 Math Story: Encourage students to draw pictures to "tell" their story if they do not have the written ability to write a short story. A quick follow up conversation will be required to know whether students are able to communicate their understanding.

REFLECTION: Reflections help increase the value of a learning experience. They allow students to link ideas and construct meaning from their experiences. Students should have opportunities to reflect on their learning at the end of every lesson. Explicit teaching about how to reflect effectively will improve the quality of student responses in this section; reflection sentence stems are available in the Connecting and Reflecting Resources page.

Data Entry

Chilliwack teachers will enter data by the end of November and by the end of May. November data entry is based on the previous year's outcomes, and is only to be completed by grades 3-7 teachers. For example, grade 4 teachers will assess their students at the beginning of the year based on the grade 3 target outcomes and using the grade 3 templates. All grades 2-7 teachers will enter data by the end of May based on the current year's outcomes.

Another unique feature of the SNAP is that students are scored by competency. You will not total or average their scores in the four competencies. Students have until the end of the school year to practice and become proficient at their grade-level learning standards, however if during your pre-assessments prior to May you have students fully proficient, you may enter their data and create learning extension opportunities for those students.

Exemplars

The exemplars on the website are intended to represent proficiency in all categories. We will be updating our exemplars on an ongoing basis. Please feel free to send in student samples that you believe clearly show student proficiency. Scan and send to joanne_britton@sd33.bc.ca.

Acknowledgements

We are grateful to the dedicated team of Chilliwack educators who crafted and piloted this assessment: *Christine Blessin, Jonathan Ferris, Kathy Isaac, Anna Lownie, Shannon McCann, Tammy McKinley, Kathleen Mitchell, Justin Moore, Kirk Savage, Paul Wojcik*

SNAP

Number Sense (0 - 100)

Name: _____

Date: _____

Draw to represent the value of the number:

Write the number in expanded form:

Count **backwards** by _____ from the number.

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

Create 3 equations that equal the number:

Write a real life example that shows the value of the number:

Count **forwards** by _____ from the number.

Show where the number belongs on the number line.

0 100

Find grading rubrics for specific criteria at snap.sd33.bc.ca

Communicating & Representing: *Drawing, description, expanded form*	Understanding & Solving: *3 equations*	Connecting & Reflecting: *Real-life*	Reasoning & Analyzing: *Skip counting & number line*
1 2 3	1 2 3	1 2 3	1 2 3

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Operations Addition SNAP

Name: _____

Date: _____

Operation: _____

Guess:

Draw:

Calculate:

Create a Math Story

Beginning	**Middle**	**End**

Communicating & Representing	Understanding & Solving	Connecting & Reflecting	Reasoning & Analyzing
1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4
Entire assessment	Draw and Calculate	Math story	Guess

Student Numeracy Assessment and Practice (SNAP)
(Adapted from ANIE 2014)

SNAP

Number Sense (0 - 1000)

Name: _____

Date: _____

Draw to represent the value of the number:

Write to describe your picture:

Write the number in expanded form:

Count **backwards** by _____ from the number.

_____

_____

_____

_____

_____

_____

_____

_____

_____

Count **forwards** by _____ from the number.

Create 3 equations that equal the number:

Write a real life example that shows the value of the number:

Show where the number belongs on the number line.

0 1000

Reflect:

Find grading rubrics for specific criteria at snap.sd33.bc.ca

Communicating & Representing: *Drawing, description, expanded form*	Understanding & Solving: *3 equations*	Connecting & Reflecting: *Real-life*	Reasoning & Analyzing: *Skip counting & number line*
1 2 3	1 2 3	1 2 3	1 2 3

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Operations Subtraction SNAP

Name: _____

Date: _____

Operation: _____

Estimate – justify your thinking:

Represent - with a sketch or drawing:

Explain your sketch:

Calculate:

Write a Real Life Example or Word Problem: _____

_____

Reflect:

Communicating & Representing	Understanding & Solving	Connecting & Reflecting	Reasoning & Analyzing
1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4
Entire assessment	Draw and Calculate	Real-life & reflection	Estimate

SNAP

Number Sense

Name: _____

Date: _____

Draw to represent the value of the number:

Write to describe your picture:

Count **backwards** by _____ from the number.

Write the number in expanded form:

Create 3 equations that equal the number:

Write a real life example that shows the value of the number:

Count **forwards** by _____ from the number.

Show where the number belongs on the number line.

Reflect:

Operations SNAP

Name: _____

Date: _____

Operation: _____

Estimate – justify your thinking:

Represent - with a sketch or drawing:

Explain your sketch:

Calculate:

Write a Real Life Example or Word Problem: _____

Reflect:

Reasoning & Analyzing

Communicating & Representing

Understanding & Solving

Reasoning & Analyzing

Number Sense Rubric
SNAP (Student Numeracy Assessment & Practice)

Competency	**Emerging** *The student demonstrates an initial understanding of the concepts and competencies relevant to the expected learning.*	**Developing** The student demonstrates a partial understanding of the concepts and competencies relevant to the expected learning.	**Proficient** *The student demonstrates a complete understanding* of the concepts and competencies relevant to the expected learning.	*Teacher notes for demonstration of understanding and applications beyond proficiency*
Communicating and Representing *Picture Box*	• Pictures do not show the value of the number • Inaccurate	• Pictures show some value in representing the number • Partially accurate	• Pictures are clearly communicated and represent the value of the number (e.g. base ten and/or symbols) • Accurate	
Describe Picture	• Description and elaboration of pictorial representation is not evident • Communication is not clear	• Partial accuracy in describing and elaborating on pictorial representation **AND/OR** partially communicated	• Accurately describes and elaborates on pictorial representation (e.g. legend, key, or words) • Clearly communicated	
Expanded Form	• Emergent understanding of the value of digits in their place values	• Partially accurate in demonstrating the value of each digit (40000 +2000+139=42139 OR 40000+100 +30+9=42139)	• Accurately demonstrates the value of each digit (e.g. 500+20+4 or five hundreds, 2 tens, and 4 ones)	
Understanding and Solving *3 Equations*	• Emergent use of operations	• Accurately uses grade appropriate operations in one or two equations	• Accurately uses grade appropriate operations in all three equations (see Exemplars for examples)	
Connecting and Reflecting *Real Life Connection*	• A real-life example is not provided or is not connection to the number	• A partial connection to a real-life example is provided (e.g. "I bought a house for $319")	• Connection to a real-life example is provided • Demonstrates understanding of the number value (e.g. 5347 leaves on a small tree shows understanding; "I live at 5347 Elm St," does not)	
Reflection	• With support, student is not yet able to reflect on their learning	• Can partially identify strengths and stretches (e.g. *"Everything was easy. Nothing was hard.")*	• With sentence frames and structure, can proficiently reflect on their learning (e.g."I feel confident with ___; ___ was challenging; my goal is___")	
Reasoning and Analyzing *Number Line*	• Emergent understanding of the placement of the number on a number line	• Partially correct estimate of placement of number on provided number line; benchmarks may be missing	• Correct estimate of placement of number on provided number line with at least three benchmarks and appropriate endpoints.	
Counting Forwards and Backwards	• Emergent understanding of place value, number sense, and/or skip counting	• Partially complete and accurate	• Complete and accurate; demonstrates understanding but may include a minor recording error	

Student Numeracy Assessment and Practice (SNAP)
SD33- Chilliwack School District
https://snap.sd33.bc.ca/

Operations Rubric
SNAP (Student Numeracy Assessment & Practice)

Competency	Emerging *Student demonstrates an initial understanding of the concepts and competencies relevant to the expected learning*	Developing *Student demonstrates a partial understanding of the concepts and competencies relevant to the expected learning*	Proficient *Student demonstrates a complete understanding of the concepts and competencies relevant to the expected learning*	Extending *Student demonstrates an insightful understanding of the concepts and competencies relevant to the expected learning*
Communicating and Representing *Entire Assessment*	• Communication (written, pictorial or symbolic) of understanding is emerging	• Communicates (written, pictorial or symbolic) partial understanding	• Communicates (written, pictorial or symbolic) clear understanding	• Communicates (written, pictorial or symbolic) insightful understanding in multiple ways
Understanding and Solving *Draw & Calculate Boxes*	• Emergent use of strategies to solve the problem and show understanding	• Strategies chosen do not lead to an accurate solution • Reasoning to solve the problem is absent	• Uses ***grade appropriate*** strategies to correctly solve the problem and show understanding	• Uses multiple strategies and/or insightful reasoning to correctly solve the problem and show understanding
Connecting and Reflecting *Real Life Example/ Word Problem*	• Emerging ability to connect mathematical concepts to real life examples	• Real life example and connections to mathematical concepts are partially developed	• Real life example and connections to mathematical concepts are evident • The example shows a clear connection to the operation	• Real life example and connections to mathematical concepts are insightful
Reflection	• With support, student is not yet able to reflect on their learning	• Can partially identify strengths and stretches "*Everything was easy; nothing was hard'*	• With sentence frames and structure, can proficiently reflect on their learning " *I feel confident with ___; ___ was challenging; my goal is___ "*	• Insightful reflection on mathematical thinking is evident
Reasoning and Analyzing *Estimate & Justify Box*	• Emerging ability to use Estimation/mental math strategies • Estimate is not yet reasonable and justification not provided	• Calculates rather than estimates *"I think it is 366 because 3x122=366"* • Strategy use is not justified *"My guess is 300 because I used mental math"*	• Reasonable estimation provided • Clearly explains strategy *"I think it is about 360 because I did 3x100=300 and 3x20=60 and added 300+60"*	• Reasonable estimation provided and insightfully explains the strategy *"I think it is about 360 because I did 3x100=300 and 3x20=60 and added 300+60 but the solution is greater than that because I rounded down""*

Student Numeracy Assessment and Practice (SNAP)
SD33- Chilliwack School District
https://snap.sd33.bc.ca/

Grade 2/3

Number Sense

Assessment Package

SNAP Assessment

First Steps in Math

Diagnostic Assessments & Learning Activities

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First Steps in Math

Curated Diagnostic Assessments and Learning Activities for Mathematics

In this section of the SD71 Number Sense Assessment Package can be used to support learning for students who have fallen into the emerging/developing categories from the SNAP assessment. Conversely, you will gain an even deeper understanding of all your students mathematical understanding if they all participate in the First Steps in Math Diagnostic Activities, and all students will benefit from the First Steps in Math Learning Activities.

The First Steps in Math Assessment section is organized into two sections:

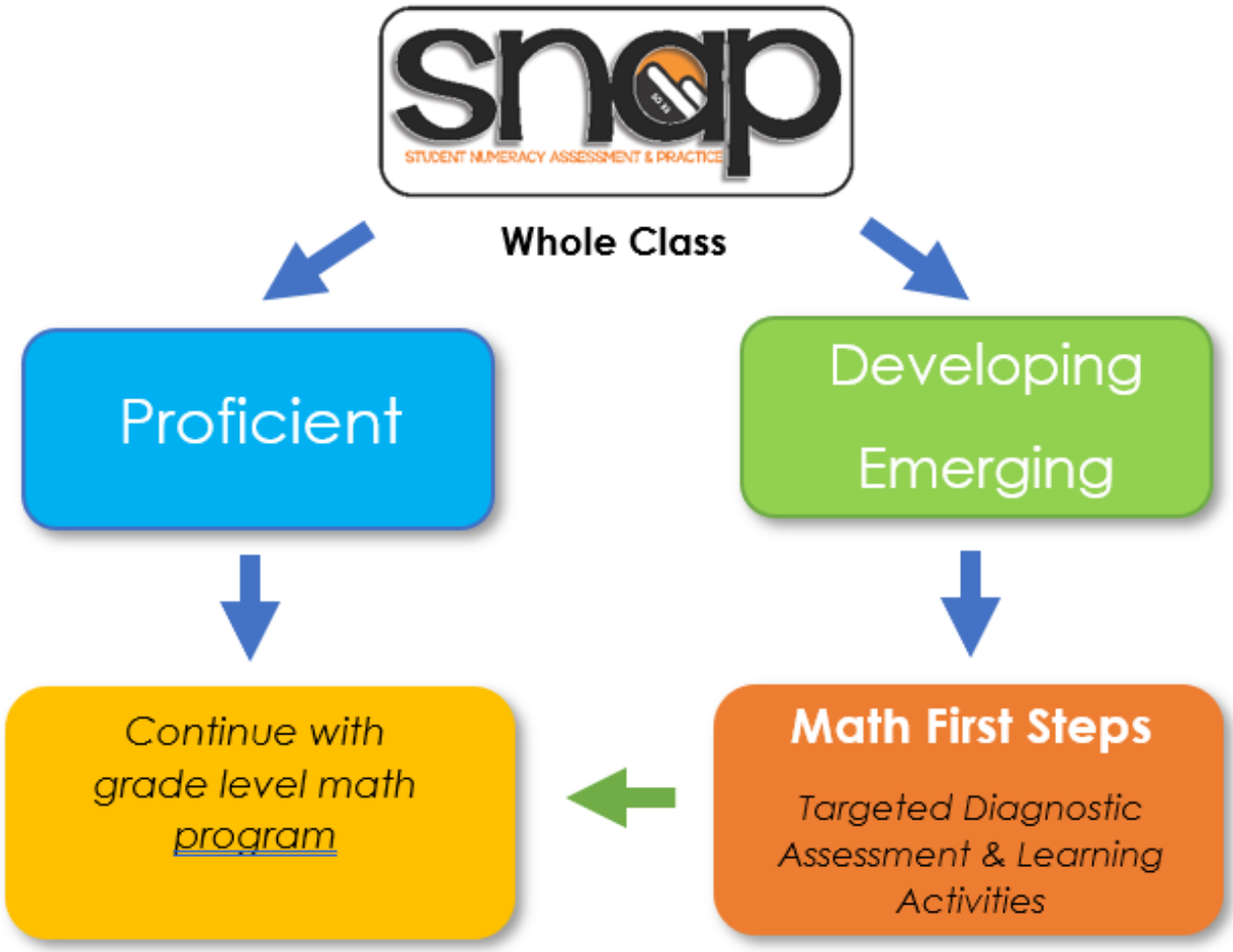
1. Diagnostic Assessments that hone in on Key Understandings of Mathematical development that students might be missing, and:

2. Learning Activities that will support learning in the key understanding areas of mathematics learning to building a strong foundation of Number Sense and relationships.

Both the Diagnostic Assessments and Learning Activities can be done one-on-one, in small groups, or whole class. Please note, in the chart on the follow page, curated grade specific diagnostic and learning activities have been hand selected to help you with your planning. If you find that there are other activities from the list that would be a better fit for your learners, please use your professional decision making to make that choice. All of the Learning Activities within a section focus on the same targeted Key Understanding, starting with the least complex to more complex. Again, all students can benefit from these activities, and they can become key components to your math program.

It is recommended that one Diagnostic and corresponding Learning Activities are focused one at a time, working with your students through those foundational key understandings at a manageable pace, revisiting multiple times. These diagnostics and activities can be woven into your existing math program and revisited every 2-3 weeks as you see fit.

All Diagnostics Assessments and Learning Activities have been curated from a larger and more comprehensive Frist Steps in Math resource that is accompanied by a 16-hour workshop to orientate and take a deeper dive into the essence of First Steps in Mathematics. If you are interested in learning more about First Steps in Math, please reach out to your School Districts Lead Teacher for Numeracy, your schools CST, or your principal.



Grade 2/3

First Steps in Math

Diagnostic Assessments

First Steps in Math

Grade 2/3 -Curated Diagnostics & Learning Activities:

NOTE: *all the following assessments and learning activities address gaps from previous grades.*

Diagnostic Assessment	Emerging Activity (EM) Suggestion	PAGE	Developing Activity (DEV) Suggestion	PAGE
Whole class/group assessments:				
Candies (NS-Key Understanding 5)	*"Place Value Beans" WITH place value tents*	p. 61	*"Wipeout"* *"Can I Have All Your…?"*	p. 58 p. 53
Up to and Through the Hundreds (NS- Key Understanding 4)	*"Number Scrolls"*	p. 57	*"Partner Number Scrolls"*	p. 57
Interview Assessments:				
Skip Counting (NS-Key Understanding 1)	*"Grouping"*	p. 46	*"Constant Addition"*	p. 47
Saying the Number Sequence by 1, 5, & 10 (NS-Key Understandings 4 & 6)	*"Partner Number Scrolls"*	p. 57	*"Backwards and Forwards"*	p. 63
Basic Facts Project – assessment cards & record sheet (COMP- Key Understanding 1)	*"Doubles"* *"Double Collections")*	p. 49 p. 50	*"Basic Facts to Ten" and Basic Facts cards & Lessons* *Cards available through printshop*	p. 51
How Did You Do It? (NS- Key Understanding 5 & 6)	*"Backwards and Forwards"* *"Number Lines"*	p. 63 p. 63	*"Adding and Subtracting"* *"Math Methods"*	p. 64 p. 62
(if struggling) see K/1 assessments				

Curated by Jeannie DeBoice 2024
Collated by Serina Allison

Diagnostic TASK

FOCUS
Understand Numbers
- Key Understanding 5

52 Candies

Years/Grades 3–7

Purpose

To explore children's understanding of the meaning of the individual digits in a two digit number when confronted by both standard and non-standard groupings of objects

Producing work samples

52 Candies

Provide each student with copies of the Candies worksheet. Make sure that students understand the sweets can be bought as single sweets or in rolls of ten.

- **Ask:** *How many candies are represented altogether?*
- **Talk** with the class about their answers. Make sure that all students agree that there are 52. Observe students as they record 52 on their page.
- **Write** '52' on the board in view of all students.
- **Point** to the '2' in the '52' on the board and say: *Use a blue pen to colour in what this part of the '52' means in the drawing.* It is important not to mention the word 'two' as you point.
- **Point** to the '5' in the '52' on the board and say: *Use a red pen to colour in what this part of the '52' means in the drawing.* It is important not to mention 'five' or 'fifty' as you point.

Diagnostic
TASK

FOCUS
Understand Numbers
- Key Understanding 5

43 Candies

Years/Grades 3–7

Purpose

To explore children's understanding of the meaning of the individual digits in a two digit number when confronted by both standard and non-standard groupings of objects

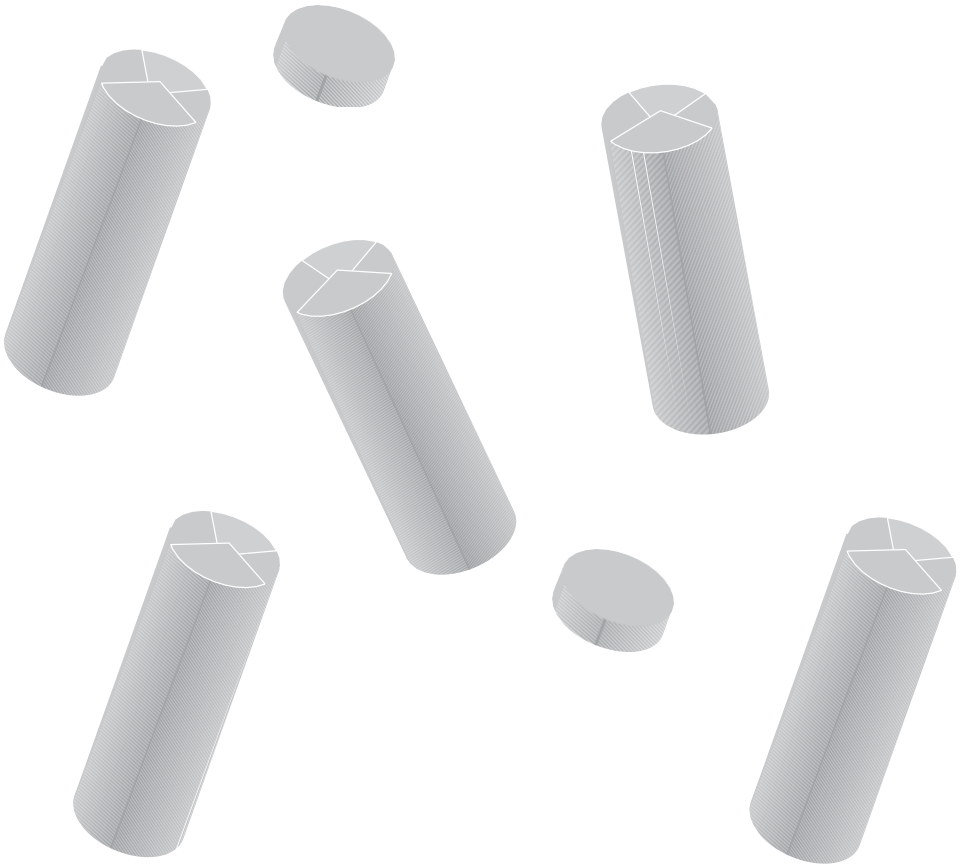
43 Candies

- **Ask:** *How many candies are represented altogether?*
- **Talk** with the class about their answers. Make sure that all students agree that there are 43. Observe students as they record 43 on their page.
- **Write** '43' on the board in view of all students.
- **Point** to the '3' in the '43' on the board and say: *Use a blue pen to colour in what this part of the '43' means in your drawing.* It is important not to mention 'three' as you point.
- **Point** to the '4' in '43' on the board and say: *Use a red pen to colour in what this part of the '43' means in your drawing.* It is important not to mention 'four' or 'forty' as you point.

Candies

Name _____ Year/Grade _____ Date _____

Candies can be bought as single candy or in rolls

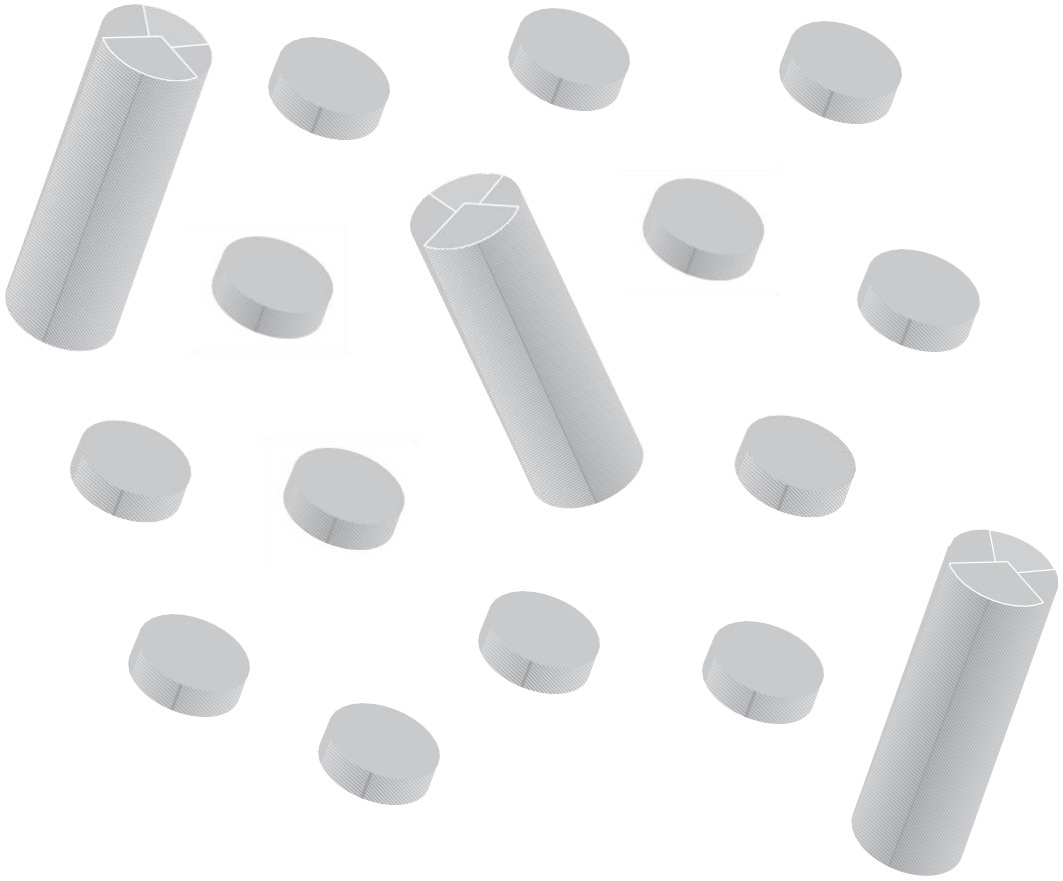


How many candies are shown here? _____

Candies

Name _____ Year/Grade _____ Date _____

Candies can be bought as single candy or in rolls of ten as shown here.



How many candies are shown here? _____

Diagnostic
TASK

FOCUS
Understand Numbers
- Key Understanding 4
- Key Understanding 5

Up To And Over 100 Grades 3–7

Purpose

To see if children know the pattern in the way we say numbers, up to and over 100

Producing work samples

Provide each child with a blank 10 x 20 grid and ask them to fill it in counting by ones, beginning at one.

Interview individual students when:

a) the student writes an incorrect number or a number incorrectly. Ask them to 'say' that part of the sequence so that you are able to hear what they actually think the pattern is.
b) the student generally experiences difficulty when working with numbers.

Up To And Through the Hundreds Grades 3–7

Purpose

To see if children know the pattern in the way we say numbers, up to and through all of the hundreds

Producing work samples

Provide each child with the provided grids and ask them to fill them in to provide the missing numbers

Interview individual students when:

a) the student writes an incorrect number or a number incorrectly. Ask them to 'say' that part of the sequence so that you are able to hear what they actually think the pattern is.
b) the student generally experiences difficulty when working with numbers.

Up To And Over 100

Name _____ Year/Grade _____ Date _____

Write the numbers to the end of the boxes.
Begin with one and count by ones to the end of the boxes.

1	2	3							

Up To And Through The Hundreds

Name _____ Year/Grade _____ Date _____

Write the numbers to the end of the boxes.
Begin at 91 and count by ones to the end of the boxes.

91	**92**	**93**							

Write the numbers to the end of the boxes.
Begin at 491 and count by ones to the end of the boxes.

491	**492**	**493**							

Diagnostic

TASK

FOCUS

Understand Numbers

- Key Understanding 1

Skip Counting

Grades 1–1

Purpose

To find out if the child knows that counting in groups gives the same result as counting by ones.

Equipment

15 small things, e.g. 15 nuts.

Producing work samples

Give a child 15 small things, e.g. 15 nuts.

Ask: *How many nuts have I given you? How did you decide that?*

Observe if the child:

- counts the 15 nuts accurately, by twos, to 14 then adds the one, stop the interview here.
- counts by ones

If the child counts by ones

Ask: *Will you get the same answer if you count by twos? Count by twos to find out.*

Observe if the child:

- keeps track of the 'twos' and what they do when they reach the remaining single nut.
- at the end of the count, notice if they call the single nut the next number in the 'twos' sequence regardless of whether there are two there or one. For example: Does the child point and say: *12, 14, 16,* **or** *12, 14, 15*. (arriving at 15 for the answer).

If by now you are sure the student knows that counting by twos gives the same result as counting by ones, stop the task. If you are still unsure then continue with the next part.

Tip out a collection of more than 50 objects, e.g. pop sticks.
Ask: *How many pop-sticks do you think are there? How could you know exactly how many are there?*

If the child: begins to count the whole collection by ones.

Ask: *How many pop sticks will there be if you count by fives?*

Skip Counting Record Sheet Date: _____

Purpose: To find out if the students knows that counting in groups gives the same
results as counting by ones.

Materials: Any collection of 15 small items.
**This task may be carried out during lessons while a student is involved in making*
and counting collections.

1. Give the student 15 small things, such as beads.
ASK: "How many beads have I given you? How did you decide that?

- Child counts to 14 by twos, then adds the one and answers '15'. STOP THE INTERVIEW THERE.
- Child counts by ones to 15 *(go on to #2)*

2. **ASK: "Will you get the same answer if you count by twos?"**

- Child answers "yes".
- Child answers "no".
- Child is unsure.

3. *Observe the child as they count & keeps track of the 'twos' and notice what they does when they reaches the single counter.*

- Child counts correctly, including 2 counters at a time and reaches 15.
- Child attempts to count by twos, but loses the 'chain', saying _____.
- Child attempts to count by twos but doesn't move the objects by twos, and says _____.
- Child is unable to count by twos at all.

*If you are still unsure whether the child **trusts** the count by twos, continue with step 4.*

4. Empty a container that holds more than 50 objects.
ASK: "How many items do you think there are?" Answer: _____
"How could you know exactly how many there are?"
Answer:_____

If child counts by ones, ask **"How many will there be if you count by fives?"**

Diagnostic TASK

FOCUS
Understand Numbers
- Key Understanding 4
- Key Understanding 6

Saying the Number Sequence by Ones and Tens

Grades 4-7

Purpose

To see if students can use the patterns in the way we say numbers to count on and back by tens from any starting point. The counting by ones gives information about KU 4. The counting by tens aspect of this task gives us information about KU 6.

Instructions

Interview: Ask the student to take over saying the number sequence where you leave off. For example, say "74, 75, 76, 77, 78, ..." Repeat this process for each of the examples below.

1. Say the number sequence going forward by ones
   a) starting from 79
   b) starting from 985

2. Say the number sequence going backwards by ones
   a) starting from 75
   b) starting from 1010

3. Say the number sequence going forward by tens
   a) starting from 180
   b) starting from 34

4. Say the number sequence going backwards by tens
   a) starting from 520
   b) starting from 146

Diagnostic
TASK

FOCUS
Understand Numbers
- Key Understanding 5
- Key Understanding 6

How did you do it?

Grades 3-7

Purpose

To see if student needs to count to solve a computation mentally, or can use partitioning based on place value or his/her own written methods

Instructions

This lesson enables students to share the strategies they use to carry out a mental computation. The lesson needs to be modelled a number of times with simple computation examples to give students practice in thinking about and recording their mental strategies.

Listen to and record student responses on the board including counting strategies if students mention them.

If the students cannot do the computation mentally, they should find an answer by using pencil and paper. Students should indicate on their page whether they worked it out mentally or used pencil and paper or materials.

Present students with either the Grades 3-5 or 5—8 Sample Learning Activity from Whole and Decimal Numbers KU 6.

Grades 3—5: Math Method *{Number Sense* Resource Book, p. 79)	**Grades 5—8: Different Strategies** *{Number Sense* Resource Book, p. 79)
62 — 23 26 + 37	Your grandpa is 84 but you only have 67 candles for his birthday cake. How many more candles do you need?

Students write the problem on their page, solve it mentally, and then record their strategy. If students cannot solve the problem mentally, tell them to work it out on the paper in a way that makes sense to them, or to use materials if they need to.

Note: Record what the students actually say and do. This will help you to reflect on what they are thinking rather than what you *assume* they are thinking.

Grade 2/3

First Steps in Math

Learning Activities

First Steps in Math

Grade 2/3 - Learning Activities & Materials

NOTE: *all the following Learning Activities address gaps from previous grades.*

Emerging Activity (EM) Suggestion	PAGE	Materials	Developing Activity (DEV) Suggestion	PAGE	Materials
"Place Value Beans" WITH place value tents	p. 61	• Counters/beans	*"Wipeout"* *"Can I Have All Your…?"*	p. 58 p. 53	• Calculators
"Number Scrolls"	p. 57	• *Paper strips /cash register tape* • *Calculators*	*"Partner Number Scrolls"*	p. 57	• *Paper strips /cash register tape*
"Grouping"	p. 46	• Counting collection items	*"Constant Addition"*	p. 47	• Calculators
"Partner Number Scrolls"	p. 57	• *Paper strips /cash register tape*	*"Backwards and Forwards"*	p. 63	• *100's charts*
"Doubles" *"Double Calculations"*	p. 49 p. 50	• *Counters / beans*	*"Basic Facts to Ten" and Basic Facts cards & Lessons*	p. 51	• *Basic Facts Cards* *Cards available through printshop*
"Backwards & Forwards" *"Number Lines"*	p. 63 p. 63	• *100's charts*	*"Adding & Subtracting"* *"Math Methods"*	p. 64 p. 62	• *100's charts*

Curated by Jeannie DeBoice 2024
Collated by Serina Allison

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first steps

First Steps in Mathematics

Number Sense

Whole and Decimal Numbers, and Fractions

Improving the mathematics outcomes of students

PEARSON

Department of Education and Training

FSMN000 | First Steps in Mathematics: Number Sense

First Steps in Mathematics: Number
Number Sense: Whole and Decimal Numbers, and Fractions

Published in Canada by Pearson Canada Inc.
26 Prince Andrew Place Don Mills, ON
M3C 2T8

Text by Sue Willis with Wendy Devlin, Lorraine Jacob, Beth Powell, Dianne Tomazos, and Kaye Treacy for Steps Professional Development on behalf of the Department of Education and Training, Western Australia

Photos: Ray Boudreau, p. 1, 3, 4, 14, 15, 20
Illustrations: Neil Curtis, Vesna Krstanovich and David Cheung

ISBN-13: 978-0-13-201885-2
ISBN-10: 0-13-201885-3

Director, Pearson Professional Learning: Theresa (Terry) Nikkel Professional Learning
Manager: Marina Pyo
Canadian Edition Consultant: Lynda Colgan
Canadian Edition Advisors: Maggie Martin Connell, Craig Featherstone, Bill Nimigon, Barry Onslow, Verna Schmidt
Project Manager: Keltie Thomas Production Coordinator:
Zane Kaneps
Art Direction: Jennifer Stimson and Zena Denchik Design: Jennifer
Stimson and David Cheung
Page Layout: David Cheung

Sample Learning Activities

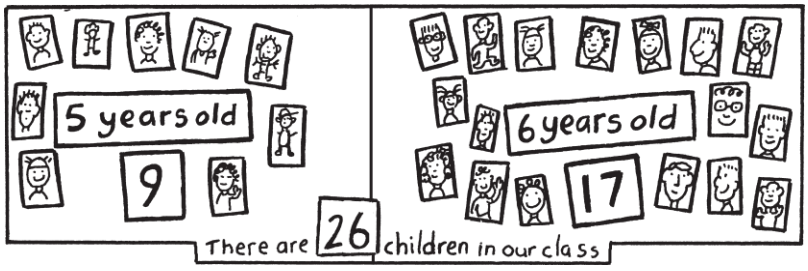
K-Grade 3: ★ ★ ★ Major Focus

Birthday Claps

Ask students to clap once for each birthday they have had. Have students link each clap with each number name as it is said.

Age Groups

Make a classroom display of students' names (photos). Arrange the names (photos) according to age groups. Have students count how many are in each group and then write number labels for the groups (*8 students are 4 years old. 15 students are 5 years old.*) As each student has a birthday, ask the student to move his or her name (photo) across to the appropriate age group. Invite all students to count how many in each group now. Ask: Which group must get smaller (bigger)?



Teeth

Vary *Age Groups*, above, by asking students to count how many in the class have (have not) lost teeth. (See Case Study 1, page 32.)

Collections

Have students make collections of a given number of things for real tasks. For example, have them choose six beads to make a necklace.

How Many?

Ask students to read number labels on storage containers to see how many things they have to get. Label shelves to show how many blocks of each type there are in the containers. During clean-up time, ask students: How many blocks have you returned so far? How many more do we need to find?

Keeping Fit

Have students decide each day (week) how many jumps and hops to include in their daily fitness routine and then record the number. Ask students to decide whether they need more or less of each action and to record this new number. Ask: How many jumps (hops) will we have today (this week)?

Labelling Collections

Invite students to count and write number labels for collections, such as buttons or keys, that they have sorted and graphed into categories of their own choosing. Have them show how they know there are more in one group than another. Ask: How do you know eight is more than seven? Would eight elephants be more than seven elephants?

Counting Cakes

Have students count a line of objects, such as playdough "cakes". Ask: Will there be the same number of cakes if we start counting from the other end? Why? Why not? Count the objects again but, this time, start with the middle object. If a student cannot do this, repeat with three objects and increase the quantity by one each time. Ask: What did you do to count all the cakes? Does it matter where you begin?

Number Trains

Have students practice the number sequence when lined up to enter or leave the classroom. Ask each student to count in turn from one to determine "how many" students are in the line. Ask: Could we find out how many are here if we count by 2s? Will we get the same number?

Biggest Number

Ask students to choose and use materials to show why 7 is less than 8 when counting a collection. Focus on the idea that the next number names a quantity which must always be one more than the number before.

Grouping

Invite students to rearrange a collection of things to make them easier to count. For example, invite students to count to see how many of them are at school today. Ask: Can we arrange ourselves so it is easy to count? Is there another way? Record the totals each time, then ask: What do you notice about how many we get every time we count? Why don't we get a different number if we start with a different person?

Choosing Equipment

Ask students to set out equipment for an art activity by referring to the number of students and collecting enough equipment for each. To begin, place one chair for each student at a table, then stand a sign on the table saying what equipment is needed, such as paintbrushes, scissors.

Different Totals

When the class is counting a collection and some students arrive at different totals for the same amount, have students consider whether or not this is possible. Ask: Could we all be right? Why? Why not?

K-Grade 3: ★ ★ ★ Major Focus

Everyday Counting

Use real counting opportunities, such as deciding how many students are going swimming or how much material is needed for an activity, to show students how counting is used by people in everyday situations.

Matching

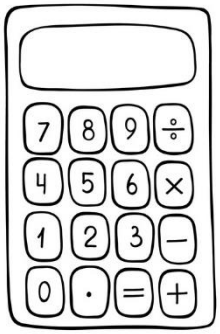
Organize the class into different-sized groups of students. Select a student in each group to collect and hand out enough sheets of paper to all members of the group. When each student collects the paper for the group, ask: Have you got the right amount of paper for your group? Will you have to come back for more paper, or will you return some sheets to me? How could you check? Focus on students' answers that include one-for-one matching and counting. Ask: Will counting help? (See Case Study 2, page 34.)

Enough for All

Invite students to suggest ways that they can check if there will be enough equipment for different numbers of people in different situations. For example, for small groups, ask a student to collect enough plastic cups for everyone at the table. For large groups, have students plan to collect enough beanbags for an activity to be held the following day. Give a reason for bringing just enough beanbags for the group. For example, say: Another class wants the leftover beanbags, so we can't take the whole box. How will we know when we have enough beanbags for our group? How could you check to see if there will be enough beanbags?

Placing an Order

Have students use plastic (playdough) food for a role play. One student could be a delivery person to whom the other students phone through an order. Encourage students to think about whether they will have enough of each thing for their group. Ask: How will the delivery person know if there is enough food for everyone? Focus students on ways to decide what is enough. When students count to find out how many they need, ask: How will counting help the delivery person bring enough food?

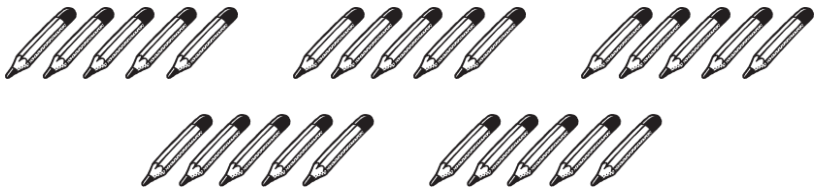


Constant Addition

Ask students to use the constant function on a calculator to count groups of things. For example, to count how many legs on five chairs, students key in 1 + = for the first leg, then = for each remaining leg. Have students record how many legs. Repeat the count by 4s for each set of legs. For example, press 4 + = for the first chair, then = = = for each successive chair. Record how many legs. Ask: Should we get the same result each time? Why? Why not?

Skip Counting a Large Collection

Have students select a number to use when skip counting a large collection. Ask: Why did you choose that number? Why wouldn't you choose to skip count by 7s or 8s? Then, extend the activity by asking students to choose a different number to re-count. Ask: Did you get the same result? Why? Why not?



Skip Counting Money

Have students skip count by 5 cents, 10 cents, and so on, up to and over $1. Then, extend the activity by asking students to skip count by $1, $5 and $10, up to and over $100.

Did You Know?

Students can use their calculators to record a count. For example, students could count the number of bicycles in the school's bicycle racks to find out how many students ride to school. To begin, students key in 1 + and then press = as they point the calculator at each additional item (1 + = = =). The calculator will display 1 and then 2, 3, 4, and so on, as students press successive equal signs. There is no need for students to press + 1 each time.

Similarly, the students can count the total number of wheels by pressing 2 followed by + = = = =, and so on, to count by 2s. The first key displays 2 and successive equal signs display 4, 6, 8, and so on. This is called constant addition.

It is also possible to do constant multiplication.

Sample Learning Activities

K-Grade 3: ★ ★ ★ Major Focus

How Many

Have students use materials, such as pinecones or bottle tops, to model an addition story involving change and then compare their answers. For example: Four butterflies are in you garden. If three more fly into your garden, how many will there be?
Ask: Are the answers all the same? What if we counted them another way? Suppose the butterflies flew around? Ask students to count the total in different ways. Record in a picture and symbols.

Counting Chickens

Ask students to model stories. For example: Mother Hen gathered four of her chickens. If three more came back, how many will there be? Continue with different examples using the same numbers until students confidently claim it will always be seven. Ask those who claim this to justify to others. When students are convinced, ask them to say it in their own words and record as a number sentence: 4 + 3 = 7.

Imagining

Have students mentally add or take away two from a small collection of objects, such as five plastic animals. Ask students to imagine that they have taken away two animals. Ask: How many animals will be left? Repeat by adding three, four, five animals. Focus students on working it out by counting on; thinking of four as two and two and counting on by twos; and thinking of five as three and two.

Addition Table

Encourage students to build up their own table pf addition facts, first to 5 + 5. Over time, build up the addition table to 6 + 6, then 7+ 7, and so on.

Number Cube Games

Play games such as this number cube game. Organize students into pairs. Give each pair three number cubes. Have students take turns to throw the cubes, add the number together and keep a running total on a calculator. During the game, ask: Which two numbers did you add together first? Why? At the end of the game (when one student reaches a total of, say, 50mor more), ask students to use number sentences to show their addition for at least one turn.

Doubles

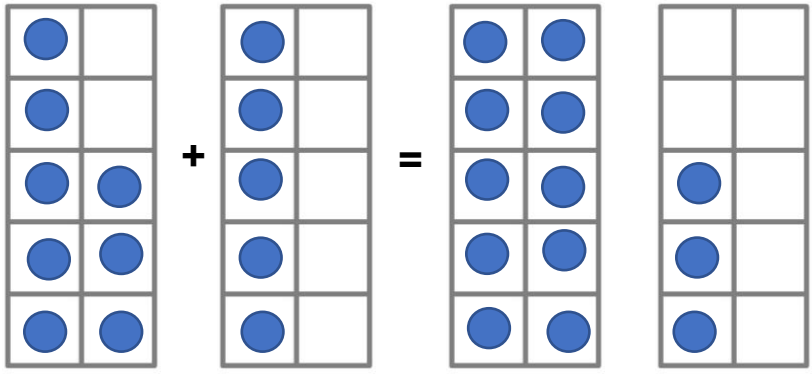
Help students develop their repertoire of known facts by building on from "doubles". For example: Have students use 6 + 6 = 12 to work out 6 + 7, 5 + 6 and so on.

Number Line

Extend the number of *Cube Games* activity on page 37. As students play the game, have them use a number line to check their mental counting on and counting back always give the same results as counting, starting the count from the same one every time.

Compensating to Ten

Ask students to use two ten-frames to find ways of breaking up numbers to calculate. For example, to add 8 + 5, students move two form the five into the frame with eight to make ten and then ass the remaining three. Extend so students can visualize the above movements.



8 + 5 = 13

Easy Calculations

Help students find an easy way of working out calculations, such as 7 + 4 or 5 + 7, using known combinations to ten. Ask them to share their strategies with the class. Extend later to include calculations with larger numbers by adding or subtracting from one of the numbers to make the other into a multiple of ten.

Double Calculations

Have students doubling collections of materials, such as beans and counters, to make "twice as many" or "two times", and tell others the results. Begin with numbers form one up to four and ask students to extend the numbers themselves. Use a diagram to show the results and describe what the groupings mean, such as two sixes and six plus six. Use double and double again for students to work out four times a given collection.

Number Combinations

Ask students to recall number combinations from contexts where they made different partitions of the same number. For example: Think about when we made necklaces. We made one with eight beads. When five were red, how many were blue? Ask them to recall the situation and "see" in their mind's eye the parts of the collection.

Basic Facts to Ten

Help students to Memorize basic facts to ten. Give each one a set of number facts cards. Ask students to work in pairs and take turns to put out three combinations of cards, such as 6 + 3, 9 + 0, and 3 + , two of which are combinations of the same number. If the partner can identify the odd card and say why, they can take the three cards. To vary the game, one student can set out two cards with equivalent totals for the partner to find another card to match the total.

Sample Learning Activities

K-Grade 3: ★ ★ ★ Major Focus

Collecting

Have students pick up a collection of 2 blocks, then 3 blocks, from a group of blocks without counting. Increase the number to 5 if students are successful at each amount.

Separating Collections

Invite pairs of students to investigate how a collection can be separated into parts. For example, students take turns to drop a collection of beans and tell their partners the number of pieces in the separated parts of the collection. For example, a collection of eight beans may fall into groups of 4 and 3 and 1. Students then record these groupings both pictorially and numerically. Compile each pair's results into a class chart to use in future lessons.

Die Combinations

Organize students into pairs. Give each pair two dice. Have students take turns to roll the dice and then say how many dots just by looking. Ask: How many dots are on the first die? (*2*) How many dots on the second die? (*3*) How many dots altogether? (*5*) Have students use calculators to keep progressive scores. The first student to reach a given number, such as 50, could be the winner. Later, extend the activity to include three dice.

How Many?

Flash small groups of things, such as leaves or stones, to students. Ask them to say how many at a glance without one-to-one counting.

Flash Cards

Show students a flash card with, for example, seven things in groupings of 5 and 2. Ask: How many things are there? What helped you see how many there are?

Snap

Organize students into pairs. Have students use adhesive dots or drawings to make sets of cards with groupings of up to 6 dots randomly placed. When playing, have students say the number of dots on the cards if there is a match. Later, add number cards where students match numbers to dots arranged in domino patterns up to 10.

Straws

Have each student hide five straws under his or her desk, some in each hand. Then, invite all students to show one hand. Ask those students with the same number of straws revealed to stand up and compare their groupings. Ask: Do you all have the same number of straws in the other hand? Have students record their groupings for five straws, then try different arrangements. Focus on the part-whole relationships of the numbers. Repeat the activity and gradually include more straws.

Five Little Monkeys

Use story contexts to help students group numbers in an organized way. For example, invite students to draw a large tree and a small tree. Then give students 5 monkey templates, such as photos cut from magazines, to move from tree to tree. Begin with 1 monkey in the small tree and 4 in the large tree. Say: There are 4 monkeys in the first tree and 1 monkey in the second tree. Move 1 monkey so that there are 3 monkeys in 1 tree and 2 monkeys in the other. Then, ask: Are there still 5 monkeys altogether? Repeat the activity for combinations of 2 and 3, 1 and 4, and 0 and 5.

Hands Up

Have two students face each other, then clap their hands three times before holding up 5 to 10 fingers. Have them show all the fingers on one hand and some extra fingers on the second hand. Together, students say how many fingers are held up altogether.

Game: Can I Have All Your

1. Play this game with a partner.

2. Each Person should enter a 3-didget number into a calculator, keeping it hidden from his or her partner.

3. Partner A asks: "Can I have all your 8s [or 3s, or 2,s, etc.]?"

4. a) If Partner B does not have that digit, they say, "You can have nothing."
   b) If Partner B has that digit, they say, "You can have 80 [or 808, or 88, depending on place value]."

5. Partner B subtracts the value. Partner A adds the value.

6. Partner B now asks: "Can I have all your…" and play continues.

7. The winner is the first to reach 1000 or make his or her opponent reach 0.

Extensions: Play the game using decimals

Sample Learning Activities

K-Grade 3: ★ ★ Important Focus

Ordinal Numbers

Have four students stand in a line in front of the class. Decide with the class which student is first, second, third and fourth. Then, ask students to close their eyes while the line is reshuffled. Ask: Who is first now? Who is in the third place? Is this place always second?

Hungry Caterpillar

Read *The Very Hungry Caterpillar* by Eric Carle with the class. Invite students to recall the order of the events in the story. Ask questions, such as: What did the caterpillar eat on the third day (Wednesday)?

Everyday Events

Ask students to make an ordered list of jobs they need to complete before school (*First, I get out of bed. Second, I have breakfast …*). Ask: What do you do fifth in the morning?

Order in the Classroom

Brainstorm classroom routines in which students would benefit from establishing and using an order. For example, setting up a classroom computer schedule. Have students list the users in order and number each one. Ask: What position are you? How many students get to use the computer before you?

Patterns

In groups, have students discuss the order of objects, such as different-coloured blocks, tiles, or beads used in a pattern sequence they have made. Help students ask each other questions, such as: What will your tenth (twentieth) piece look like? Then, invite students to continue their patterns to find out and record which positions have the same coloured block (*My second, fourth, sixth, eighth and tenth blocks are all blue triangles*).

Sports Stars

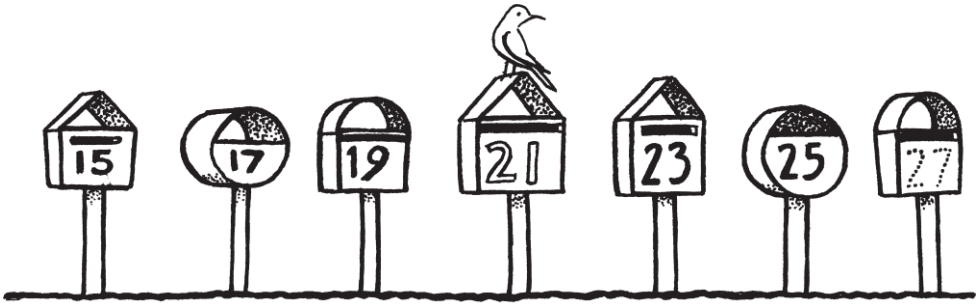
Show a photograph of a sporting team to students. Invite each student to say the number on their favourite sporting star's uniform. Ask: Is a person with a number 1 jersey more important than a person with a number 16?

Phone Numbers

Ask each student to write, then hold up his or her telephone number for others to read. Ask: Does anyone have the same phone number as you? Do they have part of your phone number? Which part? What does that part mean?

Number Hunt

During a walk around the school or neighbourhood, encourage students to find out where numbers are used and what they are used for. Pay particular attention to room, apartment, house and bus numbers, as well as car licence plates. Have students decide whether the numbers describe order. Ask: What do these numbers tell us about these rooms (apartments, houses, cars, buses)?



Sample Learning Activities

K-Grade 3: ★ ★ ★ Major Focus

Jack-in-the-Box

Have students play games that involve chanting numbers. Initially, ask students to count into the teens. Then, have students choose a number between 10 and 20. In unison, the class counts up to the chosen number and one student, playing the role of "Jack", jumps up in the air. Similarly, have students count down from a selected number to one, then the class calls out "Blast off!"

Numbers and Objects

Display collections of 13 to 19 objects that are found in the classroom, such as pencils, and their matching number. Arrange the objects in ways that highlight the way the number is said. For example, 14 pencils can be arranged as:

Numbers and Actions

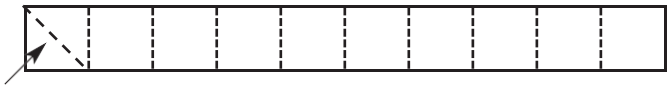
Ask students to count aloud matching the count to the rhythm of actions. For example, skips with a rope, hops with a hoop, or catches of a ball.

Number Line

Invite students to make a number line around the room in chunks of numbers, such as 0 to 10. Begin with the range 0 to 10, then add 11, 12, 13 to 19, 20, 21 to 29, and so on. Ask: What sounds the same about the new numbers? How does each new number sound different from the others? Before counting from 1, focus students' attention on when the number pattern sounds different, such as from 12 to 13 and from 19 to 20. Ask: What comes after 13 (14, 15)? What parts of the twenties sound the same as the thirties?

Number Scrolls

Invite students to generate decade and hundred number sequences by using the constant function on a calculator and to record the sequences on cash register tape. Have students fold strips of cash register tape into equal-sized squares as shown below and record one number per square. Then ask students to read, say, predict and verify the numbers from the calculator display.



Counting Sequences

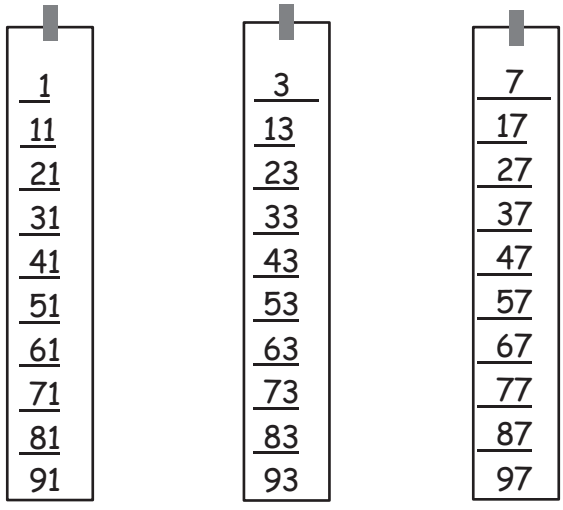
Ask the class to form a line. Beginning at 1, have students say in turn the next number in the counting sequence, going down the line and then back again. Over time, begin the count at, say 8, 18, 25, 30, 48, 95 to extend the count into the larger numbers.

Biggest Number

Select students to write the biggest number they know at the top of a display board. Ask each student: What is one more? Write the new number beneath the first. Then, have students add to the sequence each day and say the new number. Ask: Can this number sequence come to an end?

Partner Number Scrolls

Invite students to make number scrolls from cash register tape as in *Number Scrolls*, above, and use the constant function on their calculators to fill them in. Have students start from any number between 1 and 9 and constantly add 10. Then, organize students into pairs. One student reads aloud the numbers in the chart vertically by tens while his or her partner keys in the agreed starting number, such as 3, and constantly adds 10. Encourage the student with the chart to call stop at any time, then ask: What number will be next? Have students check the calculator display against the chart.



Sample Learning Activities

Grades 3-5: ★ ★ ★ **Major Focus**

Patterns in Numbers

Ask students to make their own 100-chart, arranging the numbers in whatever number of rows and columns they like. Have students use their chart to look for patterns in the numbers. Then, ask students to quickly find a number, such as 67 or 42. Show students a 10 x 10 100-chart (See Appendix: Line Master 5) and ask: What changes from one row to the next? Why? What changes in the other charts? Why? In which chart is it easier to find particular numbers? Why? Have all students make a chart for their personal use. Encourage students to extend their chart over time.

1	13	25
2	14	26
3	15	27
4	16	
5	17	
6	18	
7	19	
8	20	
9	21	
10	22	
11	23	
12	24	

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21			

Number Cube Rolls

Ask pairs of students to take turns to throw a number cube and record results in a row on squared paper, which is 5 squares wide. Have students choose which square to enter each digit in order to make the largest possible number. When both students have made a five-digit number, the player with the largest number chooses a different rule, such as *Make the lowest number or the number closest to 50 000*.

Wipeout

Play with the whole class. Enter a number, such as 256, into a calculator. Ask: How can we make the 5 a 0? (*Subtract 50.*) Why did you do that? What number have we got now? Eliminate the 2. Try larger numbers when students are ready. Later, have students play Wipeout in pairs, taking turns to give each other instructions. Encourage students to try larger numbers, such as 946 256.

10 Times as Great

Organize students into pairs. Invite students to use their calculators to find out what numbers are 10 times as great as the given numbers, such as 30, 172, 109, 200, 210, 4550. Say: Can you see a pattern? Try to explain to your partner why that happens. What will 10 times 7568 be? Test it and see.

Counting in Hundreds

Ask students to use constant addition on a calculator to count in hundreds. Have them predict which number will come next, then press = to verify. Ask: How many hundreds did you put in to make 900? How many hundreds are in 1000 (2000)?

Multiplying by 10

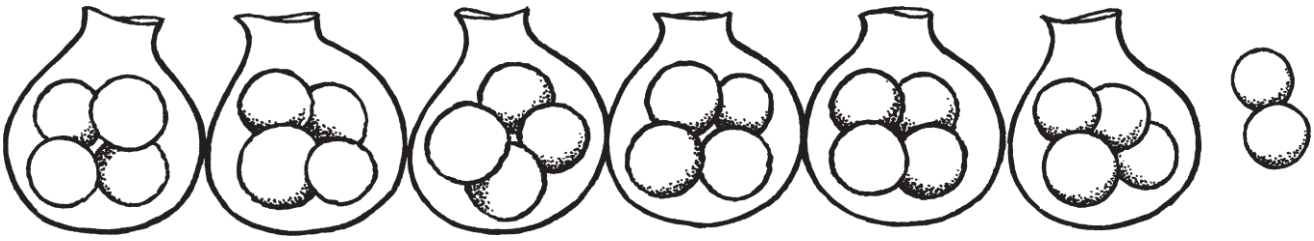
Have students predict the effect of multiplying a number by 10. Use the overhead projector calculator and begin with any single digit. Ask: If we multiply this number by 10, what will the number be? If we multiply by 10 again, what will the number be? How many tens in 100 (1000)?

Three-Digit Numbers

Ask students to use grid paper to draw a diagram that shows the size of each of the digits in a three-digit number, such as 888. Ask: How do you know you have the size right for each of the digits? How many times as big is the second 8 than the first? Later, have students represent the size of the digits in other three-digit numbers, such as 256, without using grid paper.

Marbles

Have students explain the meaning of the digits in a numeral using materials that are deliberately not grouped in standard ways—that is, not in tens—such as 26 marbles. For example, students put out 6 bags of 4 marbles and 2 more marbles. Ask: How many marbles? Have students write down how many. Record the correct answer on the board. Point to one digit and ask students to show their partner the number of marbles it refers to. Point to another digit and repeat. Repeat this activity with other collections that are not grouped in tens, for example, 3 bundles of 10 Popsicle sticks and 13 singles.

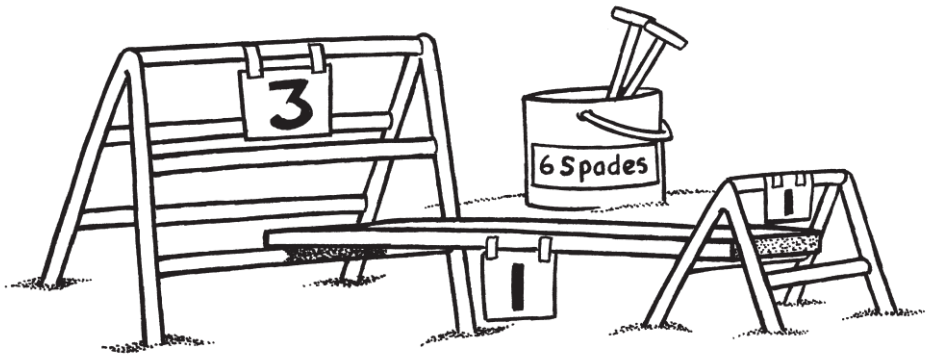


Sample Learning Activities

K-Grade 3: ★ ★ Important Focus

Number Labels

Ask students to write temporary number labels to show, for example, how many things are stored in each container in the classroom, or how many students can play on a piece of play equipment at any one time. Students could write new labels when the other labels need replacing. Ask: Which number do you need to write? Where can you find one to copy?



Bingo

Give students practice in recognizing number symbols. To begin, students could use "Bingo" cards that include the numbers 0 to 10. Gradually extend the numbers to include the "teens" and "decades", for example: 25, 52, 34, 43, 91, 19.

Matching

Organize students into pairs and have them play card games to match numbers to collections. For example, give each pair a set of cards. Half of the cards show different collections of 0 to 10 objects; the rest of the cards show a digit between 0 and 9. The game ends when all the card pairs have been matched. Extend the numbers into the teens and beyond as students are ready.

Next Number

Ask students to read aloud the numbers on their calculators as they use the constant function to count. Stop students at 9, then ask: What number will be next? Check to see if you are correct. What is different about 9 and 10? Has the calculator used these single numbers before? Use students' responses to discuss the number of digits and the difference the place makes.

Place-Value Beans

Invite students to count a handful of beans and record how many. Point to the digit representing the decade. For example, point to the 3 in 34 and ask students to find that number of beans. Focus on what the 3 in that place means. Repeat for the 4.

Reach My Number

Ask students to make their own place-value kits. On a blank sheet of 81/2 x 11 paper, have them rule three columns, then add the headings (from left to right) "Hundreds", "Tens", "Ones". Have students agree on a target number. Then, have students take turns to roll a ten-sided number cube (See Appendix Line Master 6), or use a spinner, to obtain a number from 0 to 9. Have them use their place-value kits to keep score. When students reach the target number, such as 45, ask: How would the groups look if the number was 54? Which parts would be different?

Expanded Notation

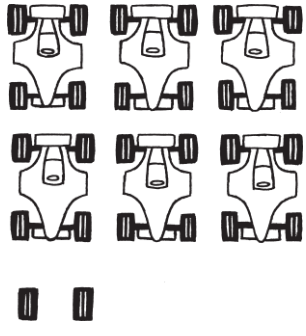
Invite students to read and record numbers as expanded notation (*28 is 2 tens and 8 ones*). Have them also write numbers from expanded notation shown in place value order as well as reversed order. Students should know that **"8 ones and 3 tens" or "2 tens and 8 ones" are both 28.**

Many students are able to tell you which is the tens column and which is the ones column and can readily write 82 as 80 + 2. However, they may still have an uncertain grip on place value and not really understand that the 8 in 82 means 8 tens. Such students often cannot sustain a place-value interpretation of numbers when confronted with non-standard groupings of things. This is a key distinction between the Partitioning and Factoring phases in students' understanding of how numbers work.

Diagnostic Activity

Ask a student to do the following activity. Look at the picture of cars and wheels. How many wheels are there? (Most students correctly write 26.) Point at the 2 and say: Use a red pen to show me this. Then, point at the 6 and say: Use a blue pen to show me this.

Did You Know?



Sample Learning Activities

Grades 3-5: ★ ★ Important Focus

Teams

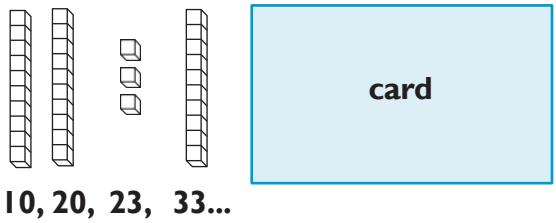
Discuss with students how to arrange three classes into teams so that the total number of students could be easily counted. Ask: Do groups of 10 make it easier to count? Can you see groups of 10 in other groups? (5s, 12s)

Regrouping

Have pairs of students decide who has the most toothpicks. Each pair takes two handfuls of toothpicks and groups them to make counting easier. Then, they count the toothpicks before regrouping them in a different way and re-counting. Ask: Which grouping made it easier to count? Did you get the same number? Why? Why not? Is it easier to see "how many" in the groups of three or groups of ten? Why?

Counting On

Arrange some Base Ten Blocks, so there are some ones, some tens, then some more ones, more tens, and so on. Invite students to count on by 1s and 10s to say how many Base Ten Blocks altogether. Cover the blocks with a piece of card and gradually uncover the materials as the count proceeds. Rearrange the blocks and ask students to re-count. Ask: Why is the total the same? Extend the activity to include hundreds.



Math Methods

Present an operation horizontally on the board, such as 62 – 23. Allow time for students to calculate the answer in their heads, then ask them to explain what they did. Record different methods for calculating on the board and draw out how most methods break up the numbers. Ask: Why did you break up the numbers in that way? Why did you put those two numbers together first?

Trading

Organize students into groups of two to four. Have them play trading games with Base Ten Blocks and a ten-sided number cube (See Appendix: Line Master 6), where, for example, 7 is worth 7 hundreds. Players collect their hundreds and trade into the thousands. At the end of a given period, students record their totals and say how many hundreds they have thrown. Ask students to count by hundreds to check.

Backwards and Forwards

Have students use 100-charts (See Appendix: Line Master 5) to count forwards and backwards by 10s. Ask: What happens to the number in the tens place each time you move forwards (backwards)?

Number Lines

Invite students to construct their own number lines to show the same movements as in the previous activity. Ask: At what number does your number line need to start? Does it need to show all of the numbers between the counted numbers? Why? Why not?

Representing Numbers

Have students use materials grouped into tens, such as linking cubes, Popsicle sticks, washers, Base Ten Blocks, to construct as many representations of a given number as possible. For example, *37* could be represented as *3 tens and 7 ones, 2 tens and 17 ones, 1 ten and 27 ones, or 37 ones*. Ask students to record their representations and justify each by showing how their groups of materials link to the representations. Extend the activity to include three-digit numbers.

Word Problems

Ask students to solve word problems using standard and non-standard place-value groupings. For example, ask: What are the possible ways 45 candies could be sold if they can be bought as singles or as rolls of ten? Have students illustrate the groups they make and use numbers to record the different ways. For example, 45 ones or 3 tens and 15 ones.

Arranging Objects

Have students work out the different ways they could buy 95 chocolates or cupcakes if they come in boxes of 5, 10, 15, 20, 50. Ask students to record their ideas and select the arrangement of 95 chocolates or cupcakes that they would prefer to buy for their family. Have students represent and justify their choices.

Sample Learning Activities

Grades 5-8: ★ ★ ★ **Major Focus**

Grouping Objects

Ask students to work out all the different ways they could buy 795 stickers if the stickers come loosely as singles, boxes of 10 packages of 10, or boxes of 10 packages. Have students record the possible ways with diagrams and/or numbers and then later with numbers alone.

Flexible Numbers

Have standard and non-standard place-value partitions for two- or three-digit numbers, such as 61, 312, 454, on separate cards. Invite students to select and record the cards that can be used to represent each number. For example, for 312, they might select:

3 hundreds	1 ten	2 ones
2 hundreds	11 tens	2 ones
2 hundreds	10 tens	12 ones

Adding and Subtracting

Have students add and subtract numbers by visualizing a 100-chart. For example, show students a 100-chart for a few minutes and then remove it from view. Ask: What number is below 43? How do you know? What number is three to the right of 72? How do you know? You are at 34. Go right two places and up three. Where are you now? You are at 68. How do you get to 75? Then, have students describe the jumps needed to calculate 24 + 39 and 83 – 47.

Calculating

Extend *Adding and Subtracting*, above, by having students make up a 1000- chart, with 1 to 100 along the top row, 101 to 200 on the second row, and so on. Ask similar questions as students use the chart to work out and explain their jumps.

Leap Along a Number Line

Have students make jumps of 1, 10 or 100 on a number line to calculate 423 + ☐ = 632, or 891 – 674 = ☐.

Different Strategies

Invite students to use partitioning and place value to solve problems mentally. Present students with a problem, such as: Your grandfather is 84, but you have only 67 candles. How many more do you need? Give students time to work the problem out in their heads and write down the answer. Ask: How did you do it? Write on the board the different ways students solved the problem. Discuss with students which methods they prefer and why.

Partitioning Numbers

Have students work in pairs to partition numbers to help them do calculations. For example, to calculate 99 x 27, students might see 99 as 100 – 1 and think,
That's one hundred 27s less one 27, then jot down:

2700
– 27

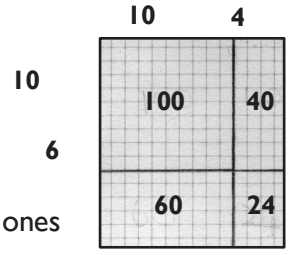
To calculate 4 x 27, students might think, *Four groups of 20 and four groups of 7*, and jot down the partial products on paper:

80
+ 28

Later, for 34 x 27, students might think, *That's thirty 27s add four 27s*, leading to something like the standard algorithm.

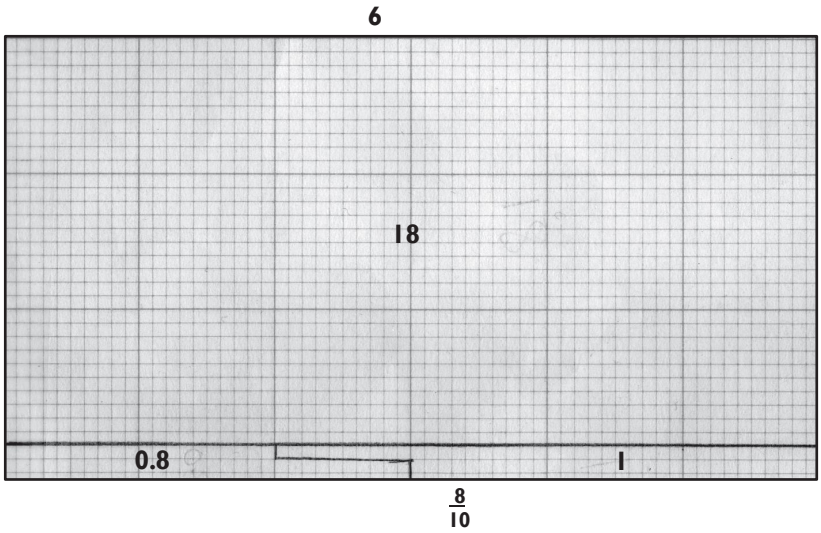
Grid Partitions

Invite students to explore ways of breaking up numbers for multiplication calculations. For example, represent 16 x 14 using grid paper and find an easy way of breaking up the grid to help work out the total. Then, ask students to share the various partitions and decide which ones make calculating easier.



Decimals

Have students use 2-mm grid paper (See Appendix: Line Master 3), with a 10 x 10 square representing one whole, to represent and calculate with decimals. For example, to calculate 6 x 3.3, a student might draw around the squares showing 6 groups of 3.3 and show that 6 x 3 = 18 and write: *18. 6 x 3 tenths is 1 and 8 tenths that's 1.8. Add 18 + 1 + 0.8 to reach a total of 19.8*. Invite students to compare the different ways they used the grid to break up the numbers to work it out.



BACKGROUND NOTES

Learning Basic Facts

Students should not be expected to try to memorize facts they do not understand. Equally, however, understanding where the basic facts come from and having worked them out for themselves is NOT enough to enable students to remember them. Students usually do need some drill with number facts if they are to be able to readily recall them. What is needed is a rational rather than a rote approach to learning the basic facts.

Addition Facts

Students might first discover and record the addition combinations to ten, convincing themselves that particular number facts "always work". They can often do some of these by counting in their mind's eye (three and two more) and/or quickly checking using their fingers, as well as by using materials. They should develop organized lists showing the numbers that fit together (part-part-whole) to make five or eight or ten. Once there is a meaningful basis for these "facts", students need focused practice on remembering them—small amounts at frequent intervals.

Meanwhile, many of the combinations to 20 should be being established using materials and diagrams. Students should be using mental arithmetic to extend the facts they already know and remember. For example: *I don't know 7 + 5 but it is*  *, which is two more than 5 + 5, so it must be 12; 8 + 6 is like two sevens so it must be 14.*

Addition Facts

+	1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10	11
2	3	4	5	6	7	8	9	10	11	12
3	4	5	6	7	8	9	10	11	12	13
4	5	6	7	8	9	10	11	12	13	14
5	6	7	8	9	10	11	12	13	14	15
6	7	8	9	10	11	12	13	14	15	16
7	8	9	10	11	12	13	14	15	16	17
8	9	10	11	12	13	14	15	16	17	18
9	10	11	12	13	14	15	16	17	18	19
10	11	12	13	14	15	16	17	18	19	20

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Once the combinations to 20 have been discovered and recorded, students should be introduced to the use of a two-way table to record the facts (See Appendix: Line Master 18).

Students will need to learn how to read the table and should investigate patterns in it. They should note the sums on either side of the diagonal are in a sense "the same" and that the number of facts to be remembered is almost halved when we use the commutative property of addition (55 instead of 100). The "double numbers" on the diagonal (2, 4, 6, 8, 10, ...) are helpful in a lot of contexts and students' attention should be drawn particularly to this sequence of numbers.

Multiplication Facts

The usual approach to learning multiplication facts is to learn to chant through the multiplication facts in order. Students learn the two times "table", the three times "table", then the four times "table", then the fives, and so on. While students do need to memorize the basic multiplication facts, learning them by chanting tables is not a particularly helpful approach, for the following reasons.

- Firstly, many students who have learned their "tables" in this way have difficulty remembering the facts without chanting through the table. Hence, it hinders the development of instant recall rather than helping it.
- Secondly, setting out the multiplication facts in columns and learning each set of tables separately masks the commutative property. Therefore, many students who *do* know, say, six fives, do not relate it to five sixes and have to remember almost twice as many facts as they need to.
- Thirdly, other patterns, such as that six times is double three times, are masked, which also increases the memory load for students.

It is likely to be much easier for students to remember basic facts if they practise them in clusters that help work them out. A possible sequence could be as follows:

Build up the facts to 5 x 5

Start with the twos (doubles), fours (double doubles), fives (because of the easy patterns and the links to our fingers) and then the threes. Put these in a two-way table. Commutativity reduces the 25 facts to be remembered to only 15, and if we remove the ones, there are only

ten to remember. Use the 5 x 5 table to show students that by learning just ten facts they "get" 25! Consolidate these facts and build up speed of recall with frequent short periods of practice.

X	1	2	3	4	5
1	1	2	3	4	5
2		4	6	8	10
3			9	12	15
4				16	20
5					25

Note that students may, at the same time, be able to skip count in twos, threes, fours and fives well beyond these facts. However, this requires them to work their way through the skips. This is not the same as being able to immediately recall 4 x 3, which is what focused drill should help them to do.

The ones and twos

At this stage, students could focus on the notion of "doubling" and build their capacity to readily find "double a number". (This should be linked to the diagonal of the addition table and to the notion of even and odd numbers in order to help students make connections between related mathematical ideas.)

The tens

Focus on groups of ten and counting in tens. Help students become convinced about why three tens are the same as ten threes and practise these together. There will be ten facts to remember, but the pattern makes them easy and students will readily recall them.

The squares

Many teachers also find that explorations of patterns of squares help students learn the squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100. It seems students "like" the square numbers and learn them fairly readily. Thus, facts such as 7 x 7 may be learned earlier than other related facts. Students should develop instant recall of these facts.

Students have now developed the facts from 1 x 1 to 5 x 5 and the facts involving one, two and ten and the squares. Help them make a two-way table (See Appendix: Line Master 19) in which they record the multiplication facts they now know.

Multiplication Facts

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15					30
4	4	8	12	16	20					40
5	5	10	15	20	25					50
6	6	12				36				60
7	7	14					49			70
8	8	16						64		80
9	9	18							81	90
10	10	20	30	40	50	60	70	80	90	100

There are various orders in which the remaining "facts" can be developed. Some teachers find it most helpful to move out in an ever-increasing square, so that the next cluster of facts is those to fill in the square to 6 x 6. This requires the addition of three new facts (6 x 3, 6 x 4, 6 x 5) and their "partner" facts (3 x 6, 4 x 6, 5 x 6). Then move out to 7 x 7 and so on.

An alternative sequence could be as follows.

The five facts

Build up the five facts to 5 x 10, noting the relationship to the ten facts (five eights is half of ten eights; half of eight tens is four tens or 40; so 5 x 8 is 40) and the pattern in the units digits. Add to the table and memorize.

The four facts and eight facts to 8 x 8

First build on the "doubles" or twos (2 x 1 to 2 x 10) to get the "double doubles" or fours (to 4 x 10). Add these to the table. Double the four facts to produce the eights up to 8 x 5 (if you know 4 x 3, you can double to get 8 x 3). Add these to the table. Use commutativity to work out the additional facts (if you know 8 x 3 you also know 3 x 8). When first practising these facts, give students plenty of time to work them out mentally using the doubling strategy or some other method they prefer. Gradually build up speed to get "instant recall". Use various doubling and other patterns to build up the extra eight facts (8 x 6 to 8 x 8). Practise to memorize.

The three facts and six facts

Build up the three additional three facts (3 x 6, 3 x 7, 3 x 9) and add to the table. Double these to get the sixes or use other known facts (six fours is five fours and four more). Five new facts to learn and their partners give you ten more.

The nine facts

Build on the threes and the sixes using number partitions to add the additional two nine facts and their partners. Although most are known, revisit the nines to link them to 10 – 1, so that students see that 9 x 7 is ten sevens take away seven. Initially, allow students time to do the calculation using mental arithmetic strategies and gradually help them memorize for speedy recall.

The seven facts

The seven facts are all known!

This is not the only possible sequence. The important thing is to assist students to use rational thought processes rather than rote memory to learn the facts.

Reducing the Stress

If students have reached grades 5 to 6 and are struggling to remember the multiplication facts, they may have built up some anxiety about them. Often it is worth spending time explaining how to read a 10 x 10 multiplication table and then having them systematically work through the table, crossing out those they know.

Everyone can draw a line through the "one times" and the "times one". Most know the doubles and the fives and tens. Quite a lot will know the squares. Spend some time on the commutative property. Students do not need to remember the word, but they should be able to say and understand why: *If I know 8 x 5 then I also know 5 x 8.* Show students how this reduces the number of facts to be remembered.

Remarkably, after removing the ones, twos, fives and tens and the squares, only 15 of the 100 facts remain. Emphasize that they are almost there! Most students will be able to cross off at least some of these 15.

Students can then make personal "prompt" cards for their remaining unknown facts. Have them set a personal target of, say, three to learn this week. During the week, help students work out how their three target facts relate to other facts they know. Periodically through the week, their partner should test them on the facts. When they have correctly recalled a fact, say, ten times, they cross it off their multiplication table list and store the card for later re-testing. Over

the next several weeks, their partner should test them on previous weeks' facts, as well as their targeted three for this week.

Techniques for Mental Calculation

Students need practice with a wide range of strategies for calculating mentally. Mental arithmetic is flexible, purposeful and personal so it cannot be made routine. For example: to add 99 to 125, a sensible strategy would be to add 100 and subtract one; to add 64 to 125, you might add 60 and then four; to add 64 to 96, you might add the four and then the 60.

The fact that the calculation is done mentally does not mean that the presentation is always oral. Often we add a string of numbers that we can see as when playing Scrabble. When we add the digits in each column for column addition, the sums often go well beyond the basic facts and so mental arithmetic is needed. In some cases, students (like adults) will use some informal jottings on paper to help keep track of their thinking. Recording partial answers is widely used by adults and should not be discouraged. The choice is not between fully mental approaches and standard written approaches. The goal is flexibility and efficiency rather than standardization.

Students should use place value to extend the range of calculations they carry out mentally. For example:

- Count backwards and forwards in tens: 10, 20, 30, 40, ...
- Count in tens from any starting point: 14, 24, 34, 44, ... and 53, 43, 33, ...
- Add in tens, twenties and thirties, hundreds and so on, from any starting point: 23, 43, 63, ...
- Generalize basic facts: 8 + 7 = 15 so 18 + 7 = 25, 28 + 7 = 35; 6 x 7 = 42 so 60 x 7 = 420

The properties of the operations (when multiplying several numbers, the order does not matter), the relationships between them (division is the inverse of multiplication), number partitions and place value form the basis of the following mental calculation strategies.

Use relationships (commutativity and inverses)

- Adding: order does not matter. **4 + 27** is 27 + 4, so 28, 29, 30, 31.
- Multiplying: order does not matter. **24** twos is **2** twenty-fours, so 48.
- Subtracting: thinking of an addition might help. **13 – 8**, think "eight add what is 13?"
- Dividing: thinking of a multiplication might help. **63 ÷ 9**, think "how many nines make 63?"

Compensate (partition and rearrange)

- Add: take some from one number to give to the other. **8 + 7** is 10 + 5; **68 + 37** is 70 + 35
- Multiply: take out a factor from one to give to the other. **15 x 6** is 15 times 2 times 3, so 30 times 3, so 90.
- Subtract: change the numbers by adding or subtracting the same amount. **62 – 37** is 65 – 40.
- Divide: change the numbers by multiplying or dividing by the same amount. **29 ÷ 5** is 58 ÷ 10.

Use compatible numbers and bridge

- Making change: **100 – 68**. Think "100 and what fits with 68" OR "It cost 68 cents. What's the change from $1?"
- Rearrange the order: **8 + 7 + 2** is 8 and 2 is 10 plus another 7 is 17; **68 + 27 + 12** is 68 and 12 is 80 plus 20 is 100 plus seven, so 107.
- Bridging: **9 + 4** is 9 + 1 + 3 = 10 + 3 = 13; **68 + 47** is 68 and 32 will make 100 and 15 left, so 115.

Front load (start with the biggest place)

- Bring on the tens and then the ones: **28 + 37** is 38, 48, 58 and 7 more, so 60, 65.
- Do both tens and then both ones: **68 + 37** is 90 + 15, so 100 and 5 more, so 105.

Imagine a number line

- Jump along or back: **364 – 198**: starting at 198, it takes 2 to get to 200 and another 164 to get to 364, so 166; OR starting at 364 go back 64 to 300, 100 more to 200 (so that's 164) and back 2 more to 198, so 166.

Multiply in parts (partition and multiply the parts)

- Round a number and adjust: **7 x 9** is 7 tens take away 7 ones, so 70 – 7, so 63; **99 x 6** is 600 take away six.
- Use place value partitions: **6 x 25** is 6 x 20 add 6 x 5, so 120 add 30.

Use factors

- Double, double, double: **4 x 14** is double double 14, so double 28, so 56.
- Change to a multiplication you know: **3 x 18** is 3 times 3 times 6, so 9 times 6, so 54.

- Multiply by five: **5 x 8** is 8 fives, which is 4 groups of 2 fives, 4 tens so 40.
- Multiply by fifty: **50 x 72**; 50 is half of a hundred, so half of 72 hundred, so 36 hundred or 3600.
- Multiply by twenty-five: **36 x 25**; Notice the 25 and look for 4 to make 100, so 9 x 4 x 25 or 900.
- Doubling and halving: **45 x 14** is the same as 90 x 7, so 630.