

# **“Multiplication and Division Facts: The Road to Automaticity”**

1. Progress Charts - highlight strategies student has mastered.
2. Assessment forms - Assess each section separately: Anchors first.
3. Lessons Booklet - teach lessons to whole class and/or small group.
4. Cards to copy - students progress through cards at their own rate.

## My Progress Chart for Multiplication

X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	<u>12</u>	16	20	24	28	32	36	40
5	0	5	10	15	<u>20</u>	25	30	35	40	45	50
6	0	6	12	<u>18</u>	<u>24</u>	30	36	42	48	54	60
7	0	7	14	21	<u>28</u>	35	<u>42</u>	49	56	63	70
8	0	8	16	<u>24</u>	<u>32</u>	40	<u>48</u>	<u>56</u>	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

**\*Commutative Property: All italic facts are 'flips' of the facts to the left of the Squares.**

**ANCHORS:**

x 0/0x  
 x 1/1x  
 x 2/2x  
 x 10/10  
 x 5/5x

**STRATEGIES:**

x9/9x

*Use What You Know*

Halving & Doubling  
 Squares

*Break It Up*

									90÷9	90÷10
								80÷8	<b>81÷9</b>	80÷10
							70÷7	72÷8	72÷9	70÷10
						60÷6	63÷7	<b>64÷8</b>	63÷9	60÷10
					<u>50÷5</u>	54÷6	56÷7	56÷8	54÷9	<u>50÷10</u>
				40÷4	<u>45÷5</u>	48÷6	<b>49÷7</b>	48÷8	<u>45÷9</u>	40÷10
			30÷3	36÷4	<u>40÷5</u>	42÷6	42÷7	<u>40÷8</u>	36÷9	30÷10
		20÷2	27÷3	32÷4	<u>35÷5</u>	<b>36÷6</b>	<u>35÷7</u>	32÷8	27÷9	20÷10
10÷0	10÷1	18÷2	24÷3	28÷4	<u>30÷5</u>	<u>30÷6</u>	28÷7	24÷8	18÷9	10÷10
9÷0	9÷1	16÷2	21÷3	24÷4	<b>25÷5</b>	24÷6	21÷7	16÷8	9÷9	
8÷0	8÷1	14÷2	18÷3	<u>20÷4</u>	<u>20÷5</u>	18÷6	14÷7	8÷8		
7÷0	7÷1	12÷2	<u>15÷3</u>	<b>16÷4</b>	<u>15÷5</u>	12÷6	7÷7			
6÷0	6÷1	<u>10÷2</u>	12÷3	12÷4	<u>10÷5</u>	6÷6				
5÷0	5÷1	8÷2	<b>9÷3</b>	8÷4	<u>5÷5</u>					
4÷0	4÷1	6÷2	6÷3	4÷4						
3÷0	3÷1	<b>4÷2</b>	3÷3							
2÷0	<b>2÷1</b>	2÷2								
1÷0	1÷1									

**Anchors:**  
 ÷0  
 ÷1/=1  
 ÷2/=2  
 ÷10/=10  
÷5/=5

**Strategies:**  
 ÷9  
**squares**  
 Think multiplication!

## Multiplication Facts:

*What's **your** quickest, easiest way to find these answers without your fingers?*

Name: \_\_\_\_\_

Date: \_\_\_\_\_

### **Anchors 0, 1 & 2: \_\_\_\_\_ in 1 minute**

$5 \times 0 = \underline{\quad}$

$3 \times 1 = \underline{\quad}$

$2 \times 2 = \underline{\quad}$

$0 \times 9 = \underline{\quad}$

$2 \times 4 = \underline{\quad}$

$7 \times 2 = \underline{\quad}$

$5 \times 9 = \underline{\quad}$

$10 \times 0 = \underline{\quad}$

$6 \times 1 = \underline{\quad}$

$1 \times 4 = \underline{\quad}$

$2 \times 7 = \underline{\quad}$

$4 \times 2 = \underline{\quad}$

$8 \times 2 = \underline{\quad}$

$2 \times 6 = \underline{\quad}$

$1 \times 8 = \underline{\quad}$

$2 \times 3 = \underline{\quad}$

### **Anchors 5 & 10: \_\_\_\_\_ in 1 minute**

$4 \times 5 = \underline{\quad}$

$9 \times 5 = \underline{\quad}$

$5 \times 3 = \underline{\quad}$

$8 \times 5 = \underline{\quad}$

$6 \times 10 = \underline{\quad}$

$10 \times 8 = \underline{\quad}$

$5 \times 5 = \underline{\quad}$

$10 \times 7 = \underline{\quad}$

$3 \times 10 = \underline{\quad}$

$5 \times 2 = \underline{\quad}$

$5 \times 6 = \underline{\quad}$

$5 \times 8 = \underline{\quad}$

$5 \times 5 = \underline{\quad}$

$7 \times 5 = \underline{\quad}$

$4 \times 10 = \underline{\quad}$

$9 \times 10 = \underline{\quad}$

### **Break It Up: \_\_\_\_\_ in 1 minute**

$4 \times 3 = \underline{\quad}$

$3 \times 3 = \underline{\quad}$

$3 \times 6 = \underline{\quad}$

$7 \times 9 = \underline{\quad}$

$6 \times 4 = \underline{\quad}$

$8 \times 4 = \underline{\quad}$

$4 \times 4 = \underline{\quad}$

$9 \times 6 = \underline{\quad}$

$9 \times 8 = \underline{\quad}$

$4 \times 7 = \underline{\quad}$

$6 \times 6 = \underline{\quad}$

$8 \times 8 = \underline{\quad}$

## Division Facts:

*What's **your** quickest, easiest way to find these answers without your fingers?*

Name: \_\_\_\_\_

Date: \_\_\_\_\_

### **Think Anchors 0, 1, & 2: \_\_\_\_\_ in 1 minute**

$5 \div 0 = \underline{\quad}$

$3 \div 1 = \underline{\quad}$

$4 \div 2 = \underline{\quad}$

$9 \div 0 = \underline{\quad}$

$8 \div 2 = \underline{\quad}$

$14 \div 7 = \underline{\quad}$

$5 \div 5 = \underline{\quad}$

$10 \div 0 = \underline{\quad}$

$6 \div 6 = \underline{\quad}$

$4 \div 1 = \underline{\quad}$

$14 \div 2 = \underline{\quad}$

$8 \div 4 = \underline{\quad}$

$16 \div 8 = \underline{\quad}$

$6 \div 1 = \underline{\quad}$

$8 \div 1 = \underline{\quad}$

$6 \div 3 = \underline{\quad}$

### **Think Anchors 5 & 10: \_\_\_\_\_ in 1 minute**

$20 \div 5 = \underline{\quad}$

$80 \div 10 = \underline{\quad}$

$15 \div 3 = \underline{\quad}$

$40 \div 5 = \underline{\quad}$

$50 \div 10 = \underline{\quad}$

$45 \div 5 = \underline{\quad}$

$25 \div 5 = \underline{\quad}$

$35 \div 7 = \underline{\quad}$

$20 \div 10 = \underline{\quad}$

$10 \div 2 = \underline{\quad}$

$30 \div 6 = \underline{\quad}$

$40 \div 8 = \underline{\quad}$

$45 \div 5 = \underline{\quad}$

$35 \div 5 = \underline{\quad}$

$70 \div 10 = \underline{\quad}$

$90 \div 10 = \underline{\quad}$

### **Think Multiplication: \_\_\_\_\_ in 1 minute**

$12 \div 3 = \underline{\quad}$

$20 \div 4 = \underline{\quad}$

$18 \div 6 = \underline{\quad}$

$49 \div 7 = \underline{\quad}$

$24 \div 4 = \underline{\quad}$

$21 \div 7 = \underline{\quad}$

$16 \div 4 = \underline{\quad}$

$42 \div 6 = \underline{\quad}$

$56 \div 8 = \underline{\quad}$

$24 \div 8 = \underline{\quad}$

$36 \div 6 = \underline{\quad}$

$64 \div 8 = \underline{\quad}$

# Strategies for Multiplication Facts

**ANCHORS (x0/0x, x1/1x, x2/2x, x10/10x):** Anchors are the easiest ones for students to learn first. (x1), the (x2) and (x10) will build the foundation for the next 2 strategies: Use What You Know & Break It Up.

**x1/1x: (Identity Factor)** Students should see **x1** is the **identity factor**, simply reflecting the other factor, showing its *identity*.

**x2/2x: (Doubler)** Students should see **x2** as 'doubling'. Remind them they learned the *doubles* in Primary grades, and how  $4 + 4 = 2 \times 4$

**x10/10x: (Tack on the zero)** Students should see how **x10** is like **x1** but goes up by a decade. *Don't let them say you just 'add' a zero! Adding a zero doesn't affect the answer, but **tacking on** a zero does!* The best way to teach this is through arrays - Compare an array of  $1 \times 9$  to one that's  $10 \times 9$ . See how much bigger the second one is (10 times bigger, of course).

## **USE WHAT YOU KNOW (x5/5x and x9/9x):**

**X5/5x: NOTE – ONCE 5'S ARE MASTERED, THEY ARE ALSO CONSIDERED 'ANCHORS',** as they are needed in *Break It Up* strategy. Students may be able to efficiently count by 5's so quickly, *they don't need special study of the x5's.* However, if they can't, they can think **x10 cut in half** (of course, exploring this first with arrays)

$$5 \times 6 = 10 \times 6 \text{ cut in half}$$

**x9/9x:** Students can use their anchor of x10 (what they know) with their knowledge of arrays. For example:

$$10 \times 5 = 50, \text{ so } 9 \times 5 \text{ is one less row of } 5.$$

**BREAK IT UP (including Squares):** This strategy handles all the rest of the facts, including squares. It will also be the basis of teaching multi-digit multiplication, so its' worth doing deeply and well!

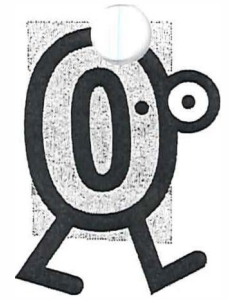
# *Starting the Fact Cards:*

## Strategies for Multiplication Facts

- The following pages are possible lessons to help your students understand the strategies on the multiplication cards.
- *Not every lesson needs to be done with every student! Some will only need to be done 1-1 with a student who struggles with that strategy (especially the earlier ones –  $\times 0$ ,  $\times 1$ ,  $\times 2$ )*
- *Some strategies can be done with the whole group – but you don't have to wait for the whole group to be at that place with the cards.*
- *Some teachers like students to keep a record of each strategy by gluing the arrays they cut into a notebook.*

# The Multiplication Strategies:

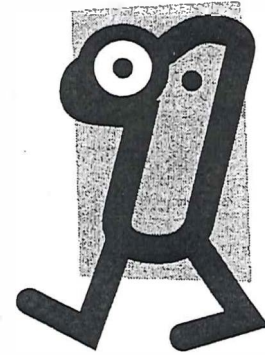
$x0/0x$



- Start with a **real-life problem**: “Pop comes in 6-packs, but you have no packs ( $0 \times 6$ ). How much pop do you have?” AND “You have 6 bags to fill with candy, but you have no candy! ( $6 \times 0$ ) How much candy is in the bags?”
- (For a few children, this is a hard concept, so you’ll most likely work 1-1 with them, not the whole class.
- \*The whole class should have experienced this idea in context, in lessons previous to this fact mastery practice.)
- *Help them come to the generalization that  $x0/0x$  is always zero.*



$$x1/1x$$



- Start with a **real-life problem**:  
“You put 5 cookies on each plate and you have just one plate. How many cookies? (1x5). How many do you have?”
- **AND** “ You have 1 cookie on each plate and 5 plates. (5x1)How many cookies do you have?”
- *Move to icon cards when you see evidence that they are beginning to answer without the models/context.*
- \*Some children will do this easily – others will need a longer time with manips – hence the need to **individualize**.
- Be sure to refer to the large multiplication chart you are building in your classroom – notice how 1x6 (1 by 6) and 6x1 (6 by 1) are related (to cut the work in half!)

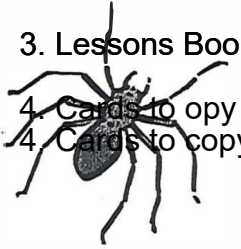
1. Progress Charts -

2. Assessment forms

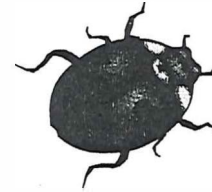
3. Lessons Booklet

4. Cards to copy

4. Cards to copy



# Doubles: (x2)



- Lead a discussion about things that come in two's – eggs (2x6) spider legs (2x4) wheels on a transporter (2x18)
- *If students have used the addition/subtraction cards, these will be familiar to them as 'Doubles'*
- Connect to arrays: have students cut different 2x arrays to relate to the previous real images.
- Have students cut all the 2x arrays and glue them onto the large class chart.
- It's also important to turn the array to show x2 to connect to the 'flip'. (You may want to have the 'flips' cut out of pink paper & glue them on to the chart.)

## Doubles!

Double 3 is the **bug double**: 3 legs on each side.

$$3 + 3 = 6$$



Double 4 is the **spider double**: 4 legs on each side.

$$4 + 4 = 8$$



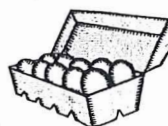
Double 5 is the **hand double**: 5 fingers on each hand.

$$5 + 5 = 10$$



Double 6 is the **egg carton double**: 2 rows of 6 eggs.

$$6 + 6 = 12$$



Double 7 is the **2-week double**:

$$7 + 7 = 14$$



2 weeks of 7 days.

Double 8 is the **crayon double**: 2 rows of 8 crayons.

$$8 + 8 = 16$$



Double 9 is the **18-wheeler double**:

$$9 + 9 = 18$$



9 wheels on each side.



# X10/10x



- This is a good place to use ‘Strings’. Strings are a series of facts that relate to each other and get kids talking about more strategies. (From Fosnot & Dolk Kit, “Investigating Multiplication & Division”)
- Write up one fact at a time on chart paper & ask students: “How did you solve that?”
- Have large grid paper arrays cut of each fact & ready onto glue on the chart paper.
- *4x10 “How did you solve this?” (If students say, “I just added a zero to the 4”, put  $0+4=4$  and ask “How did you get 40? Did you really add a zero? 40 has 4 tens, doesn’t it? Can we find them?”*
- $10 \times 4$ ,  $5 \times 10$ ,  $10 \times 5$
- $10 \times 9$ ,  $9 \times 10$ ,  $10 \times 7$ ,  $7 \times 10$
- Now have students cut & glue up the arrays for the  $10 \times$  (10 by) row on the large group mult. chart

- 1. Progress charts
- 2. Assessment forms
- 3. Lessons Booklet
- 4. Cards to copy

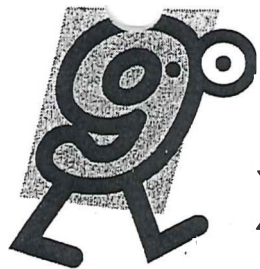


**x5/5x**

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- Another 'String': (have these arrays ready to post on chart paper titled 'x5/5x')
- $2 \times 5, 5 \times 2, 4 \times 5, 5 \times 4, 5 \times 5$
- Ask: "How did you solve these? What makes them easy/friendly?"
- "How can you use what we've done to help with the following questions?"
- $4 \times 6$  – "Which fact have we already done that would help with this one?" (4 by 5 with one more row of 4 to show 4 by 6. Sketch the new row onto the array.)
- *This is the **beginning** of developing the **distributive** principle, or 'Break It Up':*

$$4 \times 6 = (4 \times 5) + (4 \times 1) = 24$$



x9/9x


- Using another string, get the students thinking about the distributive principle again, as a strategy to help them solve  $x9$  facts.
- Post these facts, 1 at a time, and have the  $x10$  arrays prepared (you'll draw on these to solve the  $x9$  facts)

- $10x4$ , "How can you use  $10x4$  to help you solve  $9x4$ ?" (Take off a row of 4)
- $10x8$ ,  $9x8$
- $10x9$ ,  $9x9$
- $10x7$ ,  $7x9$ ,  $6x9$ .

Draw, with a felt pen, their thinking of how to use the  $x10$  arrays to solve the other  $x9$  problems.

- Students cut & glue all the  $9x$  facts on the  $9x$  row on the large group multiplication chart.



# Multiplication Fact Strategy Sheets

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## Instructions:

- Test yourself on the “Anchors” (yellow) ( $\times 0$ ,  $\times 1$ ,  $\times 2$ ,  $\times 10$ ). When you can do these in 3 seconds or less, **highlight these facts** on your Progress sheet.
- Cut out and practice the “Use What You Know” cards ( $\times 5$  &  $\times 9$ ). Practice, test, highlight on yo Progress sheet.
- Work through “Break It Up” strategies. Cut out arrays, showing your strategy through cutting the array further.
- Glue these parts to the cards. on the sheet.
- Label your thinking beside the parts of the array.
- Write the answer on the back, directly behind the array.
- When the sheets of one colour are complete, cut the cards apart & practice the facts until you have the answer in 3 seconds or less.
- Have a friend test you.
- Ask your teacher for the written test.
- Highlight those facts if you have mastery. Move on to the next set.

$2 \times 1$

$1 \times 2$



$3 \times 1$

$1 \times 3$



$4 \times 1$

$1 \times 4$



$5 \times 1$

$1 \times 5$



$6 \times 1$

$1 \times 6$



$7 \times 1$

$1 \times 7$



**X** Anchors: put answers on the back, cut apart & study.



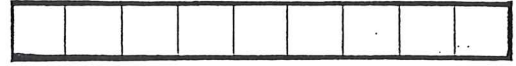
$8 \times 1$

$1 \times 8$



$9 \times 1$

$1 \times 9$



$10 \times 1$

$1 \times 10$



$11 \times 1$

$1 \times 11$



$12 \times 1$

$1 \times 12$



$1 \times 1$

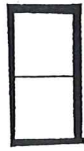
$1 \times 1$



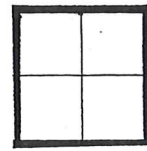
1 X Anchors

$1 \times 2$

$2 \times 1$

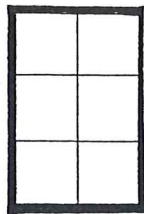


$2 \times 2$



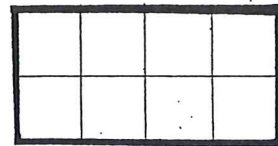
$3 \times 2$

$2 \times 3$



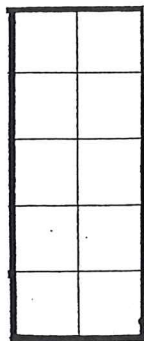
$4 \times 2$

$2 \times 4$



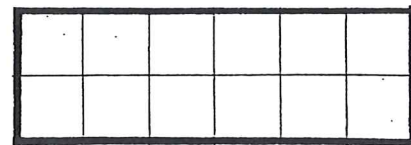
$5 \times 2$

$2 \times 5$



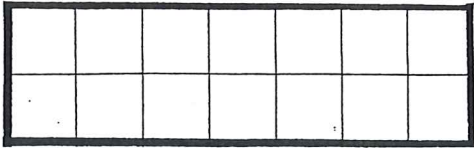
$6 \times 2$

$2 \times 6$

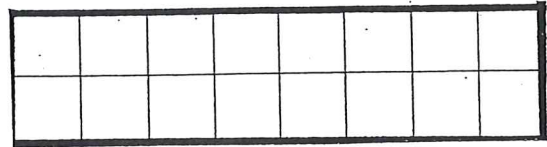


*2x/x2 Anchors*

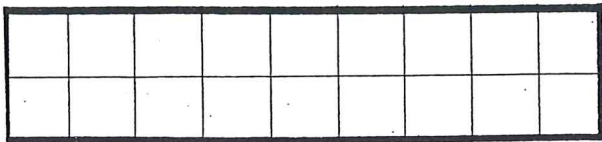
$$7 \times 2$$
$$2 \times 7$$



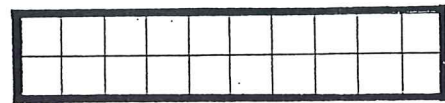
$$8 \times 2$$
$$2 \times 8$$



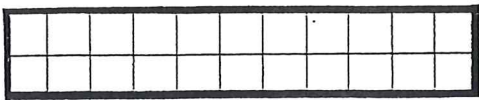
$$9 \times 2$$
$$2 \times 9$$



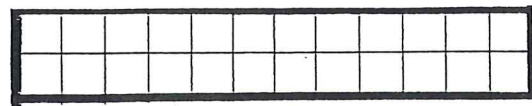
$$10 \times 2$$
$$2 \times 10$$



$$11 \times 2$$
$$2 \times 11$$

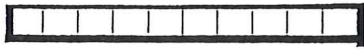


$$12 \times 2$$
$$2 \times 12$$

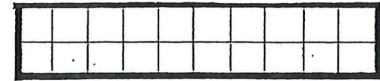


*2x/x2 Anchors*

$10 \times 1$   
 $1 \times 10$



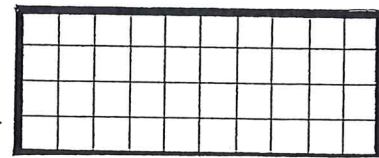
$10 \times 2$   
 $2 \times 10$



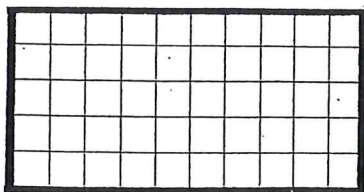
$10 \times 3$   
 $3 \times 10$



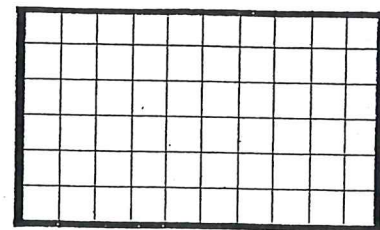
$10 \times 4$   
 $4 \times 10$



$10 \times 5$   
 $5 \times 10$

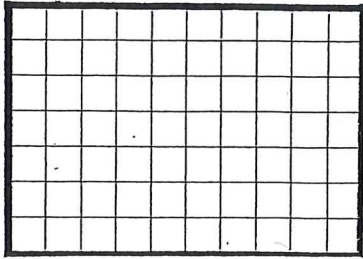


$10 \times 6$   
 $6 \times 10$

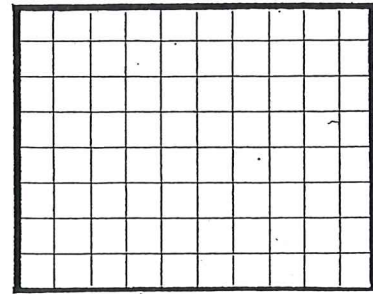


*10x/x10 Anchors*

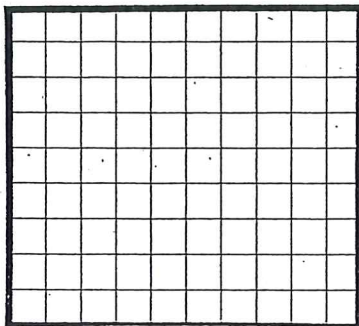
$10 \times 7$   
 $7 \times 10$



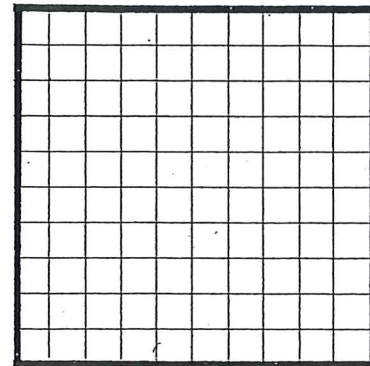
$10 \times 8$   
 $8 \times 10$



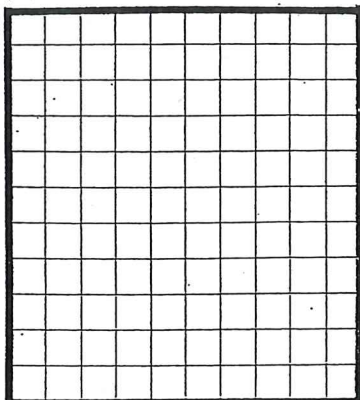
$10 \times 9$   
 $9 \times 10$



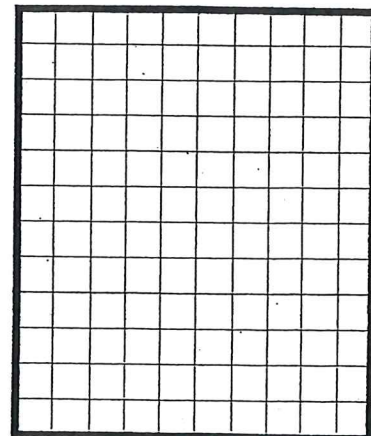
$10 \times 10$   
 $10 \times 10$



$10 \times 11$   
 $11 \times 10$



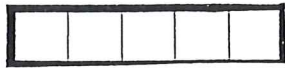
$10 \times 12$   
 $12 \times 10$



*10x/10 Anchors*

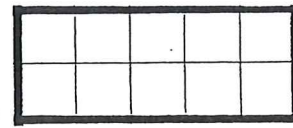
$5 \times 1$

$1 \times 5$



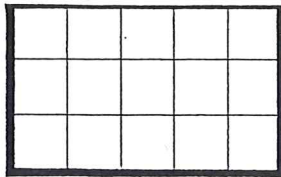
$5 \times 2$

$2 \times 5$



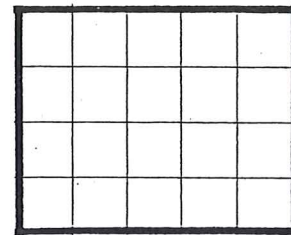
$5 \times 3$

$3 \times 5$

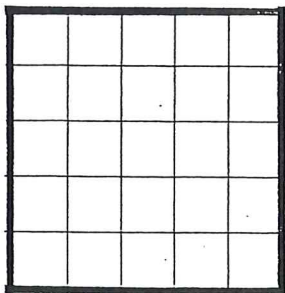


$5 \times 4$

$4 \times 5$

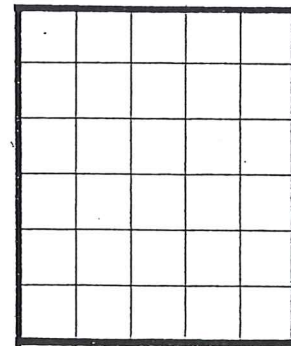


$5 \times 5$



$5 \times 6$

$6 \times 5$

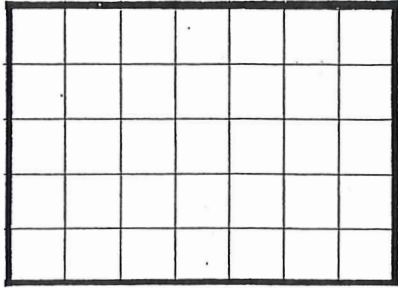


Use what you know  
x5 x9

- Put answers on back
- Cut apart & shuffle
- Study!

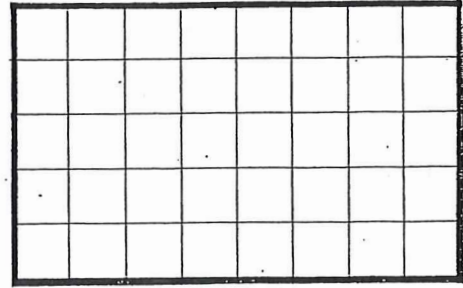
$5 \times 7$

$7 \times 5$



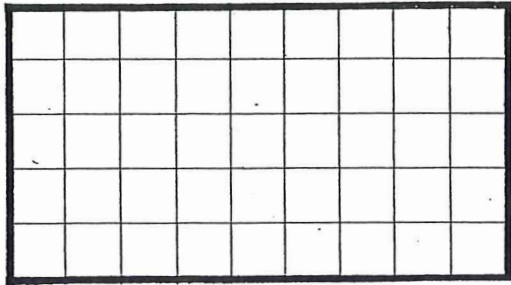
$5 \times 8$

$8 \times 5$



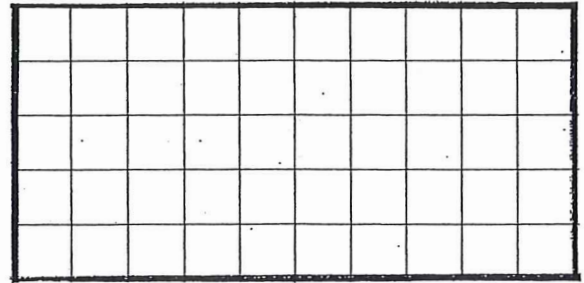
$5 \times 9$

$9 \times 5$



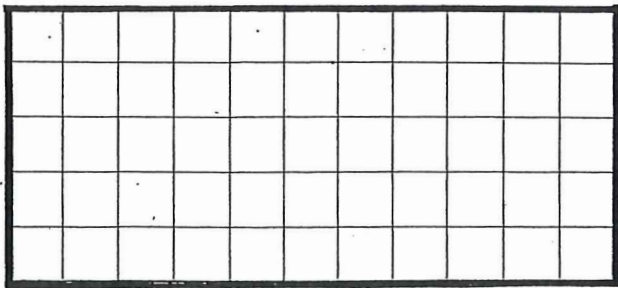
$5 \times 10$

$10 \times 5$



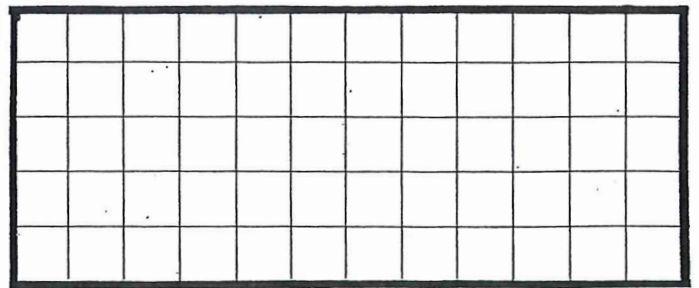
$5 \times 11$

$11 \times 5$



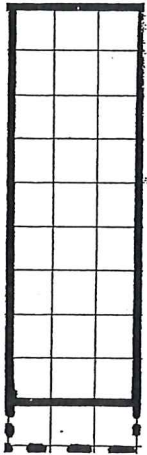
$5 \times 12$

$12 \times 5$



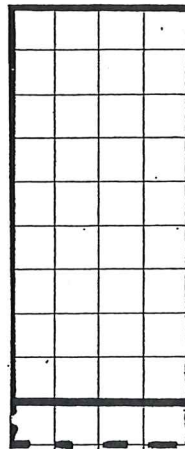
Use What You Know (X5)

$$9 \times 3$$
$$3 \times 9$$



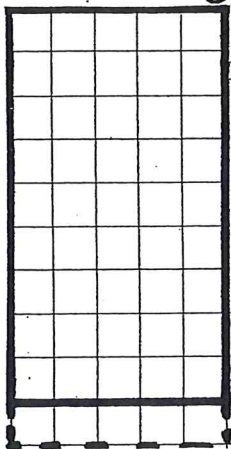
Think:  
 $10 \times 3$   
minus 3

$$9 \times 4$$
$$4 \times 9$$



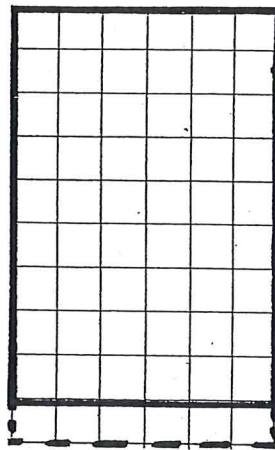
Think:  
 $10 \times 4$   
minus 4

$$9 \times 5$$
$$5 \times 9$$



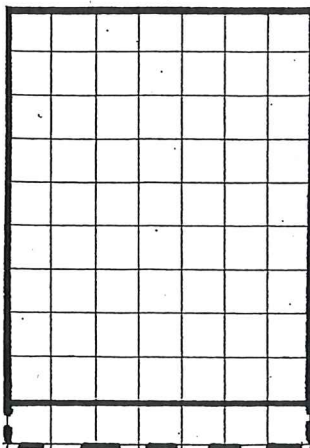
Think:  
 $10 \times 5$   
minus 5

$$9 \times 6$$
$$6 \times 9$$



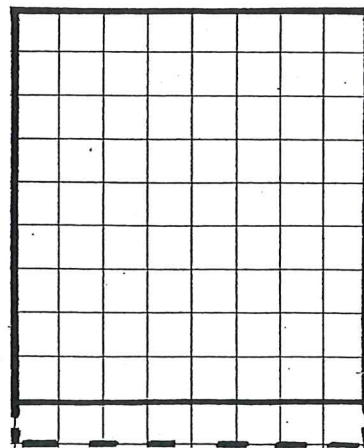
Think:  
 $10 \times 6$   
minus 6

$$9 \times 7$$
$$7 \times 9$$



Think:  
 $10 \times 7$   
minus 7

$$9 \times 8$$
$$8 \times 9$$



Think:  
 $10 \times 8$   
minus 8

Use What You Know  $\times 9$



- BREAK IT UP strategy: 1. Cut the array for the fact. 2. Break it into 2 'friendly' pieces.  
3. Glue the 2 pieces on the card. 4. Label the 2 pieces to show your strategy.  
5. Write the answer on the back. 6. Chop up the cards and practice!

$$4 \times 3$$

$$3 \times 4$$

$$5 \times 4$$

$$4 \times 5$$

$$6 \times 3$$

$$3 \times 6$$

$$6 \times 4$$

$$4 \times 6$$

$$7 \times 4$$

$$4 \times 7$$

$$8 \times 3$$

$$3 \times 8$$

- BREAK IT UP strategy: 1. Cut the array for the fact. 2. Break it into 2 'friendly' pieces.  
3. Glue the 2 pieces on the card. 4. Label the 2 pieces to show your strategy.  
5. Write the answer on the back. 6. Chop up the cards and practice!

$$4 \times 8$$

$$8 \times 4$$

$$7 \times 6$$

$$6 \times 7$$

$$6 \times 8$$

$$8 \times 6$$

$$8 \times 7$$

$$7 \times 8$$

$$6 \times 4$$

$$4 \times 6$$

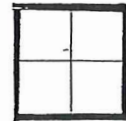
$$12 \times 3$$

$$3 \times 12$$

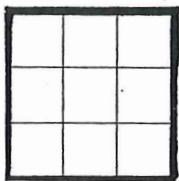
1 x 1



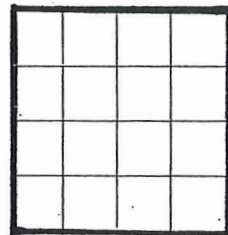
2 x 2



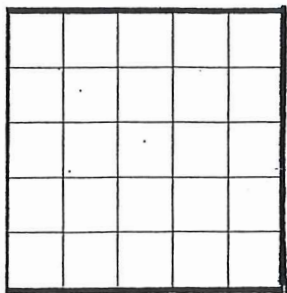
3 x 3



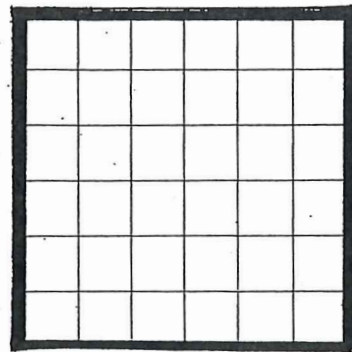
4 x 4



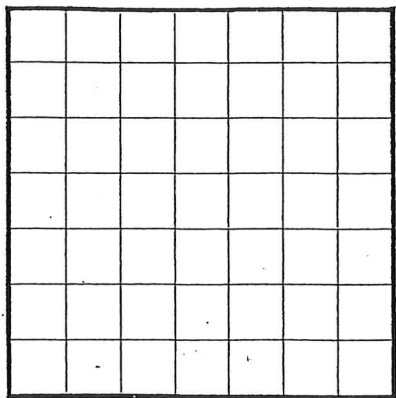
5 x 5



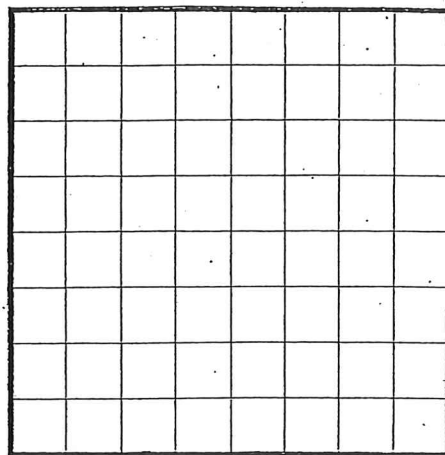
6 x 6



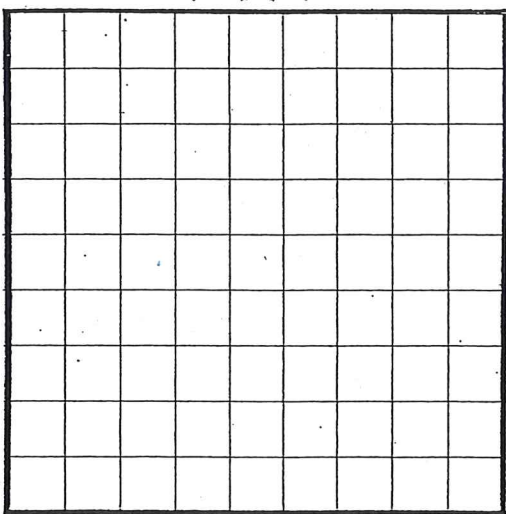
7 x 7



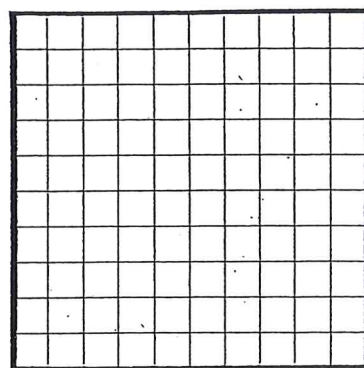
8 x 8



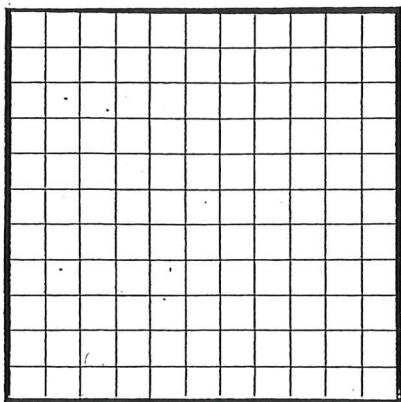
9 x 9



10 x 10



11 x 11



12 x 12

